Sample Math 115 Final Exam
Spring 2016

- The final exam is on **Thursday, May 12 at 4:30PM - 7:00PM**.
- The final exam will be held in Budig 120. The room and seat assignments for the final exam are the same as those for the common midterm exam.
- Only simple graphing calculators (TI-84 plus and below) are allowed for the common exams. You will mark your answers on both exam booklets and *provided bubble sheets*. You are considered responsible to bring pens/pencils and a calculator to the common exams. Pens or pencils will not be provided for you, and interchanging calculators will be prohibited during the exams.
- The common final exam covers
  - Chapter 2: Sections 2.1, 2.2, 2.3, 2.4, 2.5, 2.6;
  - Chapter 3: Sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7;
  - Chapter 4: Sections 4.1, 4.2, 4.3, 4.4, 4.5;
  - Chapter 5: Sections 5.1, 5.2, 5.4, 5.5;
  - Chapter 6: Sections 6.1, 6.2, 6.3, 6.4, 6.5
- The problems below, all multiple-choice with exactly one correct answer, are intended to be reasonably representative of what might appear on the actual exam, which will have **30** problems.

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1. Let $f(x) = \frac{x}{x^2+1}$ and $g(x) = \frac{1}{x}$. Then, $(g \circ f)(x)$ is

(A) $\dfrac{x}{x^2+1}$ (B) $\dfrac{1}{x}$ (C) $x + \dfrac{1}{x}$ (D) x (E) None of the above

2. Suppose that $F(x) = f(x)^2 + 1$, f(1) = 1, and $f'(1) = 3$. Find $F'(1)$.

(A) 3 (B) 4 (C) 5 (D) 6 (E) None of the above

3. The unit price p and the quantity x demanded are related by the demand equation $50 - p(x^2+1) = 0$. Find the revenue function $R = R(x)$.

(A) $\dfrac{50x}{x^2+1}$ (B) $\dfrac{50}{x^2+1}$ (C) $\dfrac{x}{x^2+1}$ (D) $\dfrac{x^2+1}{50}$

(E) None of the above

4. Find the marginal revenue for the revenue function found in Problem 3.

(A) $\dfrac{-100x}{(x^2+1)^2}$ (B) $\dfrac{1-x^2}{(x^2+1)^2}$ (C) $\dfrac{x}{25}$ (D) $\dfrac{50(1-x^2)}{(x^2+1)^2}$

(E) None of the above

5. Find $\frac{dy}{dx}$ in terms of x and y when x and y are related by the equation $x^{1/3} + y^{1/3} = 1$.

(A) $-\left(\dfrac{x}{y}\right)^{2/3}$ (B) $-\left(\dfrac{y}{x}\right)^{2/3}$ (C) $-\left(\dfrac{x}{y}\right)^{1/3}$ (D) $-\left(\dfrac{y}{x}\right)^{1/3}$

(E) None of the above

6. The derivative of the function $f(x) = \dfrac{x-3}{\sqrt{x+1}} + \sqrt{3x+4} + 3$ is

(A) $\dfrac{(x-3)\frac{1}{2}(x+1)^{-1/2} - \sqrt{x+1}}{(\sqrt{x+1})^2} + \dfrac{1}{2}(3x+4)^{-1/2} + 3$

(B) $\dfrac{(x-3)\frac{1}{2}(x+1)^{-1/2} - \sqrt{x+1}}{(\sqrt{x+1})^2} + \dfrac{1}{2}(3x+4)^{-1/2}$

(C) $\dfrac{\sqrt{x+1} - (x-3)\frac{1}{2}(x+1)^{-1/2}}{(\sqrt{x+1})^2} + \dfrac{1}{2}(3x+4)^{-1/2}$

(D) $\dfrac{\sqrt{x+1} - (x-3)\frac{1}{2}(x+1)^{-1/2}}{(\sqrt{x+1})^2} + \dfrac{1}{2}(3x+4)^{-1/2} + 3$

(E) None of the above

7. Let $f(x) = \frac{\sqrt{x+1}}{x-2}$. The domain of f is

(A) $(-\infty, 2)$ and $(2, +\infty)$ (B) $(-\infty, 2]$ and $[2, +\infty)$ (C) $[-1, 2)$ and $(2, +\infty)$

(D) $[-1, +\infty)$ (E) None of the above

8. Let $f(x) = \ln(2 - x)$. The domain of f is

(A) $(-\infty, +\infty)$ (B) $(0, +\infty)$ (C) $(-\infty, 0)$

(D) $(-\infty, 2)$ (E) None of the above

9. Evaluate: $\lim_{x \to 3}(3x^2 - 4)$.

(A) 23 (B) 5 (C) 4

(D) The limit does not exist (E) None of the above

10. Evaluate $\lim_{x \to 5} \frac{x^2-2x-15}{x-5}$.

(A) 3 (B) 8 (C) 0

(D) The limit does not exist (E) None of the above

11. Evaluate $\lim_{x \to +\infty} \frac{x^2-1}{3x^2-2}$.

(A) -1 (B) $\frac{1}{3}$ (C) $+1$

(D) The limit does not exist (E) None of the above

12. Evaluate $\lim_{x \to 1^-} f(x)$ for the function f defined as

$$f(x) = \begin{cases} e^x, & \text{for } 0 < x < 1 \\ 3, & \text{for } x = 1 \\ \ln x, & \text{for } x > 1 \end{cases}$$

(A) 0 (B) e (C) 3 (D) The limit does not exist

(E) None of the above

13. Find the horizontal asymptotes of function $f(x) = \frac{x^2}{1+4x^2}$.

(A) $y = 1$ (B) $y = \frac{1}{4}$ (C) $x = 1$

(D) The function has no horizontal asymptotes (E) None of the above

14. Find the vertical asymptotes of function $f(x) = \frac{2+x}{(1-x)^2}$.

(A) $x = -2$ (B) $x = 1$ (C) $y = 0$

(D) The function has no vertical asymptotes (E) None of the above

15. The line tangent to $y = x^2 - 3x$ through the point (1, -2) has equation

(A) $y = x - 3$ (B) $y + 2 = (2x - 3)(x - 1)$ (C) $y = -x - 1$

(D) $y - 2 = (2x - 3)(x - 1)$ (E) None of the above

16. Find an equation of the tangent line to the graph of $y = x \ln x$ at the point (1,0).

(A) $y = x + 1$ (B) $y = x - 1$ (C) $y = (x + 1)\ln x$

(D) $y = (x - 1)\ln x$ (E) None of the above

17. Find an equation of the tangent line to the graph of $y = \ln(x^2)$ at the point $(2, \ln 4)$.

(A) $y = x + 2 - \ln 4$ (B) $y = \frac{2}{x}(x - 2) - \ln 4$ (C) $y = \frac{2}{x}(x - 2) + \ln 4$

(D) $y = x - 2 + \ln 4$ (E) None of the above

18. Find an equation of the tangent line to the graph of $y = e^{2x-3}$ at the point $(\frac{3}{2}, 1)$.

(A) $y = 2e^{2x-3}$ (B) $y = 2x - 4$ (C) $y = 2x - 2$

(D) $y = 2e^{2x-3}(x - \frac{3}{2})$ (E) None of the above

19. Find an equation of the tangent line to the graph of $y = e^{-x^2}$ at the point $(1, 1/e)$.

(A) $y = -\frac{2}{e}(x + 1) + \frac{1}{e}$ (B) $y = -\frac{2}{e}(x - 1) - \frac{1}{e}$ (C) $y = -\frac{2}{e}(x - 1) + \frac{1}{e}$

(D) $y = -2xe^{-x^2}(x - 1) + \frac{1}{e}$ (E) None of the above

20. Find $\frac{dy}{dx}$ in terms of x *only* when x and y are related by the equation $\ln y = 2x - 3$.

(A) e^{2x-3} (B) $\dfrac{1}{2x - 3}$ (C) $2e^{2x-3}$ (D) $\dfrac{e^{2x-3}}{2x - 3}$

(E) None of the above

21. Find the second derivative of the function $f(x) = e^x + \ln(x^2) + x \ln 3 + 10$.

(A) $e^x + \frac{1}{x^2} + \frac{1}{3}$ (B) $e^x + \frac{1}{x} + \ln 3$ (C) 0 (D) $e^x + \frac{2}{x} + \ln 3$

(E) None of the above

22. The absolute maximum value and the absolute minimum value of the function $f(x) = \frac{1}{2}x^2 - 2\sqrt{x}$ on $[0, 3]$ are

(A) absolute min. value: $-\dfrac{3}{2}$; absolute max. value: $\dfrac{9}{2} - 2\sqrt{3}$

(B) absolute min. value: 0; absolute max. value: 3

(C) absolute min. value: 0; no absolute max. value

(D) no absolute min. value; absolute max. value: 3

(E) None of the above

23. Find the absolute maximum value and the absolute minimum value, if any, of the function $f(x) = \frac{1}{1+x^2}$.

(A) absolute min. value: 0; absolute max. value: 1

(B) absolute min. value: 0; no absolute max. value

(C) no absolute min. value; absolute max. value: 1

(D) no absolute min. value; no absolute max. value

(E) None of the above

24. Find the absolute extrema of function $f(t) = te^{-t}$.

(A) absolute min. value: 0; absolute max. value: $\dfrac{1}{e}$

(B) absolute min. value: 0; no absolute max. value

(C) no absolute min. value; absolute max. value: $\dfrac{1}{e}$

(D) no absolute min. value; no absolute max. value

(E) None of the above

25. Find the absolute extrema of the function $f(t) = \frac{\ln t}{t}$ on $[1, 2]$.

(A) absolute min. value: 0; absolute max. value: $\dfrac{\ln 2}{2}$

(B) absolute min. value: 0; absolute max. value: $\dfrac{1}{e}$

(C) absolute min. value: 1; absolute max. value: 2

(D) absolute min. value: 0; absolute max. value: e

(E) None of the above

26. Let $f(x) = \frac{1}{3}x^3 - x^2 + x - 6$. Determine the intervals where the function is increasing and where it is decreasing.

(A) increasing on $(-\infty, 1)$ and on $(1, \infty)$

(B) increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$

(C) decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$

(D) decreasing on $(-\infty, 1)$ and on $(1, \infty)$

(E) None of the above

27. Let the function f be defined in Problem 26. Find the intervals where f is concave upward and where it is concave downward.

(A) concave upward on $(-\infty, 1)$ and on $(1, \infty)$

(B) concave upward on $(-\infty, 1)$ and downward on $(1, \infty)$

(C) concave downward on $(-\infty, 1)$ and upward on $(1, \infty)$

(D) Concave downward on $(-\infty, 1)$ and on $(1, \infty)$

(E) None of the above

28. Let the function f be defined in Problem 26. Find the inflection points, if any.

(A) $(x, y) = (1, f(1))$ (B) $(x, y) = (\frac{1}{2}, f(\frac{1}{2}))$ (C) $(x, y) = (0, f(0))$

(D) No inflection points (E) None of the above

29. Let $f(x) = e^{-x^2}$. Determine the intervals where the function is increasing and where it is decreasing.

(A) increasing on $(-\infty, 0)$ and on $(0, \infty)$

(B) increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$

(C) decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$

(D) decreasing on $(-\infty, 0)$ and on $(0, \infty)$

(E) None of the above

30. Let the function f be defined in Problem 29. Find the relative extrema of f.

(A) relative min. value: 0; relative max. value: 1

(B) no relative min. value ; relative max. value: 1

(C) relative min. value: 0; no relative max. value

(D) no relative min. value ; no relative max. value

(E) None of the above

31. Let the function f be defined in Problem 29. Find the intervals where f is concave upward and where it is concave downward.

(A) concave upward on $(-\infty, 0)$ and on $(0, \infty)$

(B) concave downward on $(-\infty, 0)$ and on $(0, \infty)$

(C) concave upward on $(-\infty, -\frac{1}{\sqrt{2}})$ and on $(\frac{1}{\sqrt{2}}, \infty)$; concave downward on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

(D) concave downward on $(-\infty, -\frac{1}{\sqrt{2}})$ and on $(\frac{1}{\sqrt{2}}, \infty)$; concave upward on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

(E) None of the above

32. Let the function f be defined in Problem 29. Find the inflection points, if any.

(A) $(x, y) = (0, f(0))$ (B) $(x, y) = (-\frac{1}{\sqrt{2}}, f(-\frac{1}{\sqrt{2}}))$ and $(x, y) = (\frac{1}{\sqrt{2}}, f(\frac{1}{\sqrt{2}}))$

(C) $(x, y) = (-\frac{1}{\sqrt{2}}, f(-\frac{1}{\sqrt{2}}))$ (D) $(x, y) = (\frac{1}{\sqrt{2}}, f(\frac{1}{\sqrt{2}}))$ (E) None of the above

33. Let $f(x) = x \ln x$. Determine the intervals where the function is increasing and where it is decreasing.

(A) increasing on $(-\infty, \frac{1}{e})$ and decreasing on $(\frac{1}{e}, \infty)$

(B) decreasing on $(-\infty, \frac{1}{e})$ and increasing on $(\frac{1}{e}, \infty)$

(C) increasing on $(0, \frac{1}{e})$ and decreasing on $(\frac{1}{e}, \infty)$

(D) decreasing on $(0, \frac{1}{e})$ and increasing on $(\frac{1}{e}, \infty)$

(E) None of the above

34. Suppose that f is defined in Problem 33. Determine the intervals of concavity for the function.

(A) concave upward on $(0, \infty)$

(B) concave downward on $(0, \infty)$

(C) concave upward on $(0, \frac{1}{e})$; concave downward on $(\frac{1}{e}, \infty)$

(D) concave downward on $(0, \frac{1}{e})$; concave upward on $(\frac{1}{e}, \infty)$

(E) None of the above

35. Suppose that f is defined in Problem 33. Find the inflection points, if any.

(A) $(x, y) = (\frac{1}{e}, f(\frac{1}{e}))$ (B) $(x, y) = (1, f(1))$ (C) $(x, y) = (e, f(e))$

(D) No inflection points (E) None of the above

36. Find the derivative of function $y = x^{\ln x}$. (Hint: use logarithmic differentiation.)

(A) $y' = (\ln x)^2$ (B) $y' = \frac{2\ln x}{x} x^{\ln x}$ (C) $y' = x^{\ln x}$

(D) the derivative does not exist (E) None of the above

37. Find the derivative of function $y = 10^x$. (Hint: use logarithmic differentiation.)

(A) $y' = 10^x \ln 10$ (B) $y' = 10^x$ (C) $y' = 10^x \ln e$

(D) the derivative does not exist (E) None of the above

38. An open box is to be made from a square sheet of tin measuring 12 inches $\times$ 12 inches by cutting out a square of side x inches from each corner of the sheet and folding up the four resulting flaps. To maximize the volume of the box, take $x =$

(A) 1 (B) 2 (C) 3

(D) 4 (E) None of the above

39. A rectangular box is to have a square base and a volume of $20ft^3$. If the material for the base costs 30 cents/square, the material for the four sides costs 10 cents/square, and the material for the top costs 20 cents/square, determine the dimensions of the box that can be constructed at minimum cost. (See Fig. 1.)

(A) $x \times x \times h = 1 \times 1 \times 20$ (B) $x \times x \times h = 2 \times 2 \times 5$ (C) $x \times x \times h = 2.5 \times 2.5 \times 3.2$

(D) $x \times x \times h = 3 \times 3 \times 2.22$ (E) None of the above

40. Postal regulations specify that a parcel sent by parcel post may have a combined length and girth of no more than 108 inches. Find the dimensions of the cylindrical package of greatest volume that may be sent through the mail. (In the answers, r is the radius and l is the length.)

(A) $r \times l = \frac{35}{\pi} \times 37$ (B) $r \times l = \frac{36}{\pi} \times 36$ (C) $r \times l = \frac{37}{\pi} \times 35$

(D) $r \times l = \frac{38}{\pi} \times 34$ (E) None of the above

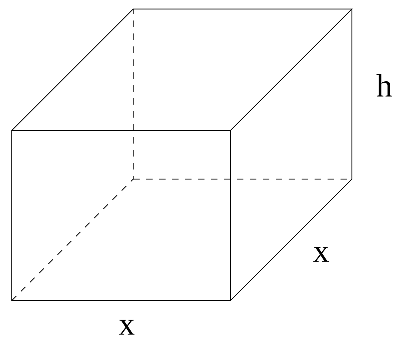

Figure 1: Problem 39.

41. It costs an artist $1000 + 5x$ dollars to produce x signed prints of one of her drawings. The price at which x prints will sell is $400/\sqrt{x}$ dollars per print. How many prints should she make in order to maximize her profit ?

(A) 1200 (B) 1400 (C) 1600 (D) 1800

(E) None of the above

42. The differential of function $f(x) = 1000$ is

(A) 1000 (B) $1000dx$ (C) 0 (D) dx

(E) None of the above

43. Use differentials to estimate the change in $\sqrt{x^2+5}$ when x increases from 2 to 2.123.

(A) 0.083 (B) 0.082 (C) 0.081 (D) 0.080

(E) None of the above

44. The velocity of a car (in feet/second) t seconds after starting from rest is given by the function

$$f(t) = 2\sqrt{t} \qquad (0 \le t \le 30).$$

Find the car's position at any time t.

(A) $\frac{4}{3}t^{3/2} + C$ (B) $\frac{4}{3}t^{3/2}$ (C) $\frac{4}{3}t^{1/2} + C$

(D) $\frac{4}{3}t^{1/2}$ (E) None of the above

45. Evaluate $\int (\sqrt{x} - 2e^x)\,dx$.

(A) $\frac{2}{3}x^{3/2} - 2e^x$ (B) $\frac{2}{3}x^{3/2} - 2e^x + C$ (C) $\frac{3}{2}x^{2/3} - 2e^x$

(D) $\frac{3}{2}x^{2/3} - 2e^x + C$ (E) None of the above

46. Evaluate $\int 2x(x^2+3)^{10}dx$. (Hint: Use substitution)

(A) $(x^2+3)^{11}+C$ (B) $\frac{1}{11}(x^2+3)^{11}+C$ (C) $(x^2+3)^{10}+C$

(D) $\frac{1}{10}(x^2+3)^{10}+C$ (E) None of the above

47. Calculate $\int_1^8 \left(4x^{1/3}+\frac{8}{x^2}\right)dx$.

(A) 49 (B) 50 (C) 51

(D) 52 (E) None of the above

48. Find the area of the region under the graph of function $f(x)=x^2$ on the interval [0, 1].

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

(E) None of the above

49. Find the area of the region under the graph of $y=x^2+1$ from $x=-1$ to $x=2$.

(A) 4 (B) 5 (C) 6 (D) 7

(E) None of the above