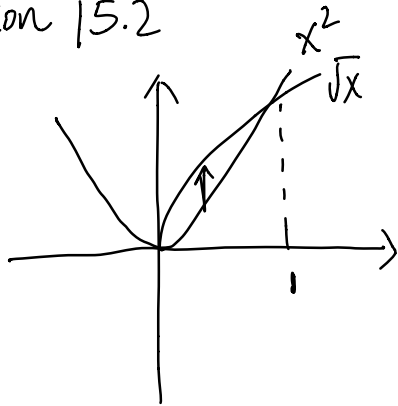


HW 7.

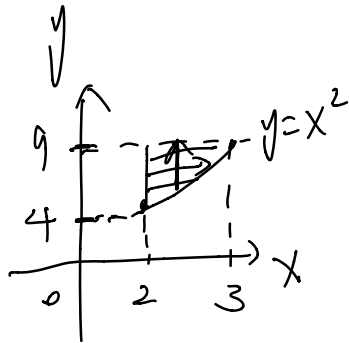
Section 15.2

21.



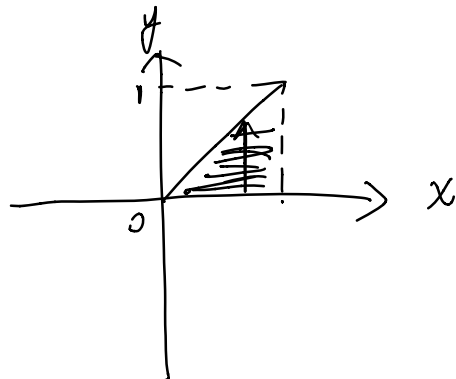
$$\begin{aligned}
 & \int_0^1 \int_{x^2}^{\sqrt{x}} (6xy - x^2) dy dx \\
 &= \int_0^1 (3xy^2 - x^2y) \Big|_{x^2}^{\sqrt{x}} dx \\
 &= \int_0^1 (3x \cdot x - x^{\frac{5}{2}} - (3x^5 - x^4)) dx \\
 &= \int_0^1 (3x^2 - x^{\frac{5}{2}} - 3x^5 + x^4) dx \\
 &= x^3 - \frac{2}{7} x^{\frac{7}{2}} - \frac{1}{2} x^6 + \frac{x^5}{5} \Big|_0^1 \\
 &= 1 - \frac{2}{7} - \frac{1}{2} + \frac{1}{5}
 \end{aligned}$$

27.

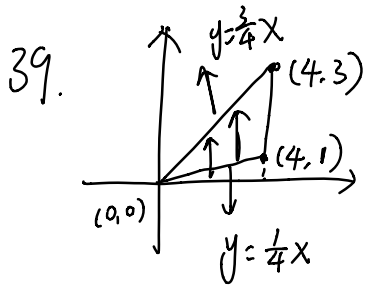


$$\int_2^3 \int_{x^2}^9 f(x,y) dy dx$$

33.



$$\begin{aligned}
 & \int_0^1 \int_0^x \frac{\sin x}{x} dy dx \\
 &= \int_0^1 (\sin x) dx \\
 &= -\cos x \Big|_0^1 = 1 - \cos 1
 \end{aligned}$$



$$\int_0^4 \int_{\frac{x}{4}}^{\frac{3x}{4}} e^{x^2} dy dx$$

$$= \int_0^4 e^{x^2} \frac{1}{2} x dx$$

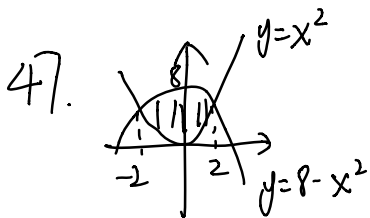
$$= \frac{1}{2} \int_0^4 e^{x^2} x dx$$

$$u = x^2$$

$$du = 2x dx$$

$$= \frac{1}{2} \int_0^{16} e^u \frac{du}{2} = \frac{1}{4} e^u \Big|_0^{16}$$

$$= \frac{1}{4} (e^{16} - 1)$$



$$x^2 = 8 - x^2$$

$$2x^2 = 8$$

$$x^2 = 4 \quad x = \pm 2$$

$$\int_{-2}^2 \int_{x^2}^{8-x^2} (16 - 2y) dy dx$$

$$= \int_{-2}^2 [16y - y^2] \Big|_{x^2}^{8-x^2} dx$$

$$= \int_{-2}^2 (16(8-x^2) - (8-x^2)^2 - 16x^2 + x^4) dx$$

$$= \frac{512}{3}$$

$$\begin{aligned}
 55. \text{ Average} &= \frac{\iint_D f(x,y) \, dA}{\iint_D (1) \, dA} \\
 &= \frac{\int_0^{\pi} \int_0^1 y^2 \sin x \, dy \, dx}{\int_0^{\pi} \int_0^1 (1) \, dy \, dx} \\
 &= \frac{\frac{1}{3} (-\cos x \Big|_0^{\pi})}{\pi} \\
 &= \frac{(-\frac{1}{3})(-2)}{\pi} = \frac{2}{3\pi}
 \end{aligned}$$

Section 15.3.

$$\begin{aligned}
 5. \int_0^1 \int_0^3 \int_0^3 (x-y)(y-z) \, dz \, dy \, dx \\
 &= \int_0^1 \int_0^3 \left((x-y)yz - \frac{(x-y)z^2}{2} \Big|_0^3 \right) dy \, dx \\
 &= \int_0^1 \int_0^3 \left(3(x-y)y - \frac{9}{2}(x-y) \right) dy \, dx \\
 &= \int_0^1 \left(\frac{3x}{2}y^2 - y^3 - \frac{9}{2}xy + \frac{9}{4}y^2 \Big|_0^3 \right) dx \\
 &= \int_0^1 \left(\frac{27}{2}x - 27 - \frac{27}{2}x + \frac{81}{4} \right) dx = \frac{-27}{4}
 \end{aligned}$$

$$7. \int_0^a \int_0^b \int_0^c (x+z)^3 dz dy dx$$

$$= \int_0^a \int_0^b \left. \frac{(x+z)^4}{4} \right|_0^c dy dx$$

$$= \int_0^a \int_0^b \left(\frac{(x+c)^4}{4} - \frac{x^4}{4} \right) dy dx$$

$$= \int_0^a b \left(\frac{(x+c)^4 - x^4}{4} \right) dx$$

$$= \frac{b}{4} \left(\frac{(x+c)^5 - x^5}{5} \right) \Big|_0^a$$

$$= \frac{b}{20} ((a+c)^5 - a^5 - c^5)$$