Hw 10.

section 13.1

29. Since $y=t$, then $z^2 = 9-y^2 = 9-t^2$

$z \geq 0 \Rightarrow z = \sqrt{9-t^2}$ from here we know

$9-t^2 \geq 0 \Rightarrow |t| \leq 3 \Rightarrow -3 \leq t \leq 3$

$$x = 2+y^2-z^2 = 2+t^2-(9-t^2)$$
$$= 2t^2-7$$

So the parametrization is

$$(2t^2-7,\ t,\ \sqrt{9-t^2}),\ -3 \leq t \leq 3$$

35. intersection $\begin{cases} x^2+y^2=1 \\ z=4x^2 \end{cases}$

Let $x=\cos t$ $y=\sin t$ $0 \leq t \leq 2\pi$

then $z = 4x^2 = 4(\cos t)^2$.

So the parametrization is

$$(\cos t,\ \sin t,\ 4(\cos t)^2),\ 0 \leq t \leq 2\pi.$$

13.2.

5. $\lim_{h\to 0} \frac{r(t+h)-r(t)}{h}$

$= \lim_{h\to 0} \frac{\langle (t+h)^{-1}-t^{-1}, \sin(t+h)-\sin t, 0\rangle}{h}$

$= \langle \lim_{h\to 0} \frac{(t+h)^{-1}-\frac{1}{t}}{h}, \lim_{h\to 0} \frac{\sin(t+h)-\sin t}{h}, 0\rangle$

$= \langle (\frac{1}{t})', (\sin t)', 0\rangle$

from definition of derivative

$= \langle \frac{-1}{t^2}, \cos t, 0\rangle$

11. $c(t)' = (t^{-1} i - e^{2t} k)'$

$= (-1)t^{-2} i - 2e^{2t} k.$

19. $\frac{d}{dt}(r_1(t)\cdot r_2(t)) = r_1'(t)\cdot r_2(t) + r_1(t)\cdot r_2'(t)$

$= \langle 2t, 3t^2, 1\rangle \cdot \langle e^{3t}, e^{2t}, e^t\rangle$

$+ \langle t^2, t^3, t\rangle \cdot \langle 3e^{3t}, 2e^{2t}, e^t\rangle$

$= (2t+3t^2)e^{3t} + (3t^2+2t^3)e^{2t} + (1+t)e^t.$

13.3.

5. length $= \int_0^3 \|r'(t)\| \, dt.$ $\quad r'(t) = \langle 1, 6t^{\frac{1}{2}}, 3t^{\frac{1}{2}} \rangle$

$$= \int_0^3 \sqrt{1+36t+9t} \, dt.$$

$$= \int_0^3 \sqrt{1+45t} \, dt$$

$$= \frac{2}{3} \cdot \frac{1}{45} (1+45t)^{\frac{3}{2}} \Big|_0^3$$

$$= \frac{2}{135} \left((1+45\times 3)^{\frac{3}{2}} - 1 \right). = \frac{544\sqrt{34}-2}{135}$$

11. $r'(u) = \langle 2u, 4u, 3u^2 \rangle$

$$s(t) = \int_0^t \sqrt{4u^2+16u^2+9u^4} \, du.$$

$$= \int_0^t u\sqrt{20+9u^2} \, du.$$

Let $v = u^2.$ $\quad dv = 2u\,du.$

$$= \int_0^{t^2} \sqrt{20+9v}\ \left(\frac{dv}{2}\right)$$

$$= \frac{1}{2}\ (20+9v)^{\frac{3}{2}} \cdot \frac{2}{3} \cdot \frac{1}{9}\ \Big|_0^{t^2}$$

$$= \left(\frac{1}{27}\ (20+9v)^{\frac{3}{2}}\ \Big|_0^{t^2}\right)$$

$$= \frac{1}{27}\left[\ (20+9t^2)^{\frac{3}{2}} - 20^{\frac{3}{2}}\right)$$

17. $r(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$$\|r(t)\| = \sqrt{9(\cos 3t)^2 + 16(\sin 4t)^2 + 25(\sin 5t)^2}$$

$t = \frac{\pi}{2}$

$\Rightarrow \|r(t)\| = 5.$