HW 11

16.1 25. $\operatorname{div}(\vec{F}) = \frac{\partial}{\partial x}(x - 2zx^2) + \frac{\partial}{\partial y}(z - xy) + \frac{\partial}{\partial z}(z^2x^2)$

$= 1 - 4zx - x + 2x^2z$

$$\operatorname{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)i - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)j + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)k$$

$\operatorname{curl}(\vec{F}) = (0 - 1)i - (2z^2x + 2x^2)j + (-y - 0)k$

$= -i - (2z^2x + 2x^2)j - yk$

29. $\operatorname{div}(\vec{F}) = \frac{\partial}{\partial x}(e^y) + \frac{\partial}{\partial y}(\sin x) + \frac{\partial}{\partial z}(\cos x)$

$= 0$

$\operatorname{curl}(\vec{F}) = (0 - 0)i - (-\sin x - 0)j$

$+ (\cos x - e^y)k$

$= (\sin x)j + (\cos x - e^y)k.$

16.2 5. $\int_0^{\pi} f(r(t))\ \|r'(t)\|\ dt.$

$= \int_0^{\pi} (1+t^2)\ \sqrt{1+1}\ dt.$

$= \sqrt{2} \int_0^{\pi} (1+t^2)\, dt$

$= \sqrt{2}\left(t+\frac{t^3}{3}\Big|_0^{\pi} \right) = \sqrt{2}\left(\pi + \frac{\pi^3}{3}\right)$

11. $\int_0^2 (16t^2)\cdot\sqrt{4+9+16}\quad dt$

$= 16\int_0^2 t^2\ \sqrt{29}\ dt.$

$= 16\sqrt{29}\int_0^2 t^2\ dt$

$= 16\sqrt{29}.\left(\frac{t^3}{3}\right)$

$= 16\sqrt{29}\ \frac{8}{3} = \frac{128}{3}\sqrt{29}.$

17. $\int_C 1 \, ds$

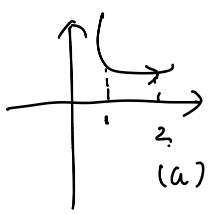
$= \int_2^5 \|r'(t)\| \, dt$

$= \int_2^5 \sqrt{16+9+144} \, dt$

$= 3 \cdot 13$

$= 39.$

The integral represents the length of the curve.

3. 

(a). $(t, \frac{1}{t})$ $1 \le t \le 2$

$F(r(t)) = (\frac{1}{t^2}, t^2)$

$dr = (1, -\frac{1}{t^2}) \, dt.$

(b). $\int_C F \cdot dr$

$= \int_1^2 (\frac{1}{t^2}, t^2)(1, \frac{-1}{t^2}) \, dt$

$= \int_1^2 (\frac{1}{t^2} - 1) \, dt$

$= -\frac{1}{t} - t \Big|_1^2$

$= (-\frac{1}{2} - 2) - (-1-1)$

$= -\frac{5}{2} + 2 = -\frac{1}{2}.$

7. $\int_0^{\pi} \langle \cos t, \sin t, t^2 \rangle \cdot \langle -\sin t, \cos t, 1 \rangle \, dt$

$= \int_0^{\pi} (-\cos t \sin t + \sin t \cos t + t^2) \, dt$

$= \frac{t^3}{3} \Big|_0^{\pi} = \frac{\pi^3}{3}$

19. $r(t) = (t, 3t) \quad t: 0 \to 1$

$\int_0^1 \langle 1+t^2, t \cdot 9t^2 \rangle \cdot \langle 1, 3 \rangle \, dt$

$= \int_0^1 \langle 1+t^2, 9t^3 \rangle \cdot \langle 1, 3 \rangle \, dt$

$= \int_0^1 (1+t^2 + 27t^3) \, dt$

$= t + \frac{t^3}{3} + \frac{27}{4} t^4 \Big|_0^1$

$= \frac{4}{3} + \frac{27}{4} = \frac{97}{12}$

25. $\int_0^1 F(r(t)) \cdot r'(t) \, dt$,

$= \int_0^1 \langle \frac{1}{\cancel{t}8t+1}, \frac{1}{t^2+1}, 1 \rangle \cdot \langle 3t^2, 0, 2t \rangle \, dt$

$= \int_0^1 \langle \frac{1}{9}, \frac{1}{t^2+1}, 1 \rangle \cdot \langle 3t^2, 0, 2t \rangle \, dt$

$= \int_0^1 (\frac{t^2}{3} + 2t) \, dt = \frac{1}{9} t^3 + t^2 \Big|_0^1 = \frac{10}{9}$.