HW 12

8. $G_u = (2u, 1, 1)$ $\quad G_v = (-2v, 1, -1)$

$$N(u,v) = G_u \times G_v = \begin{vmatrix} i & j & k \\ 2u & 1 & 1 \\ -2v & 1 & -1 \end{vmatrix}$$

$$= (-2)i - (-2u+2v)j + (2u+2v)k$$

$$\Rightarrow (-2,\ 2u-2v,\ 2u+2v)\Big|_{(2,3)}$$

$$= (-2,\ -2,\ 10)$$

$G(2,3) = (-5, 5, -1)$

tangent plane: $-2(x+5) - 2(y-5) + 10(z+1) = 0$

$$\Rightarrow -2x - 2y + 10z + 10 = 0$$

11. $\int_2^{2.1}\int_3^{3.2} ||N(u,v)||\, du\, dv$

$= \int_2^{2.1}\int_3^{3.2} \sqrt{4+4+100}\, du\, dv$

$= \sqrt{108} \times 0.1 \times 0.2 \approx 0.2078.$

17. $G(\phi,\theta) = \langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi \rangle$

$G_\phi = \langle \cos\phi\sin\theta, \cos\phi\sin\theta, -\sin\phi \rangle$

$G_\theta = \langle -\sin\phi\sin\theta, \sin\phi\cos\theta, 0 \rangle$

$G_\phi \times G_\theta = \langle \sin^2\phi\cos\theta, \sin^2\phi\sin\theta, \sin\phi\cos\phi \rangle$

$||G_\phi \times G_\theta|| = \sin\phi$

$f = \sin^2\phi\cos^2\theta$

$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin^2\phi\cos^2\theta\sin\phi\, d\phi\, d\theta$

$= \left(\int_0^{\frac{\pi}{2}} \sin^3\phi\, d\phi\right)\left(\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta\right)$

$= \left(\frac{2}{3}\right)\left(\int_0^{\frac{\pi}{2}} \left(\frac{\cos 2\theta + 1}{2}\right) d\theta\right) = \frac{\pi}{6}$

21. $G(x,y) = \langle x, y, 1-x-y \rangle$

$G_x = \langle 1, 0, -1 \rangle \qquad G_y = \langle 0, 1, -1 \rangle$

$G_x \times G_y = \langle 1, 1, 1 \rangle \qquad \|G_x \times G_y\| = \sqrt{3}.$

$$\int_0^1 \int_0^{1-x} \sqrt{3}\,(1-x-y)\,dy\,dx$$

$$= \sqrt{3} \int_0^1 \int_0^{1-x} (1-x-y)\,dy\,dx$$

$$= \sqrt{3} \int_0^1 \left(y - xy - \frac{y^2}{2} \Big|_0^{1-x} \right) dx$$

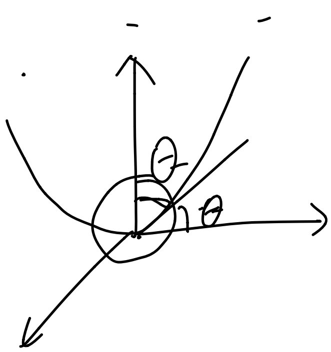
$$= \frac{\sqrt{3}}{6}.$$

27. $S = \int_0^2 \int_0^4 \|G_u \times G_v\|\,du\,dv$

$$G_u \times G_v = \begin{vmatrix} i & j & k \\ 2 & 0 & 1 \\ 4 & 0 & 3 \end{vmatrix} = -2j \Rightarrow (0, -2, 0)$$

$$S = \int_0^2 \int_0^4 2\,du\,dv = 16$$

37.



$$G(x,y) = (x, y, \sqrt{x^2+y^2})$$

$$G_x = (1, 0, \frac{x}{\sqrt{(x^2+y^2)}})$$

$$G_y = (0, 1, \frac{y}{\sqrt{x^2+y^2}})$$

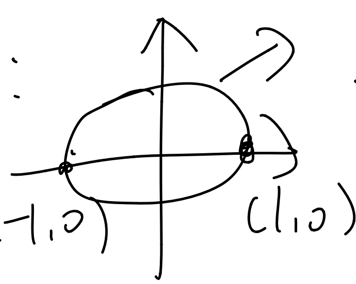
$$G_x \times G_y = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & 1 & \frac{y}{\sqrt{x^2+y^2}} \end{vmatrix}$$

$$= -\frac{x}{\sqrt{x^2+y^2}}\, i - \frac{y}{\sqrt{x^2+y^2}}\, j + k$$

$$\|G_x \times G_y\| = \sqrt{\frac{x^2+y^2}{x^2+y^2} + 1}$$

$$= \sqrt{2}$$

Bottom:



$$2y^2 + x^2 = 1$$

$$\therefore \int_{-1}^{1} \int_{-\sqrt{\frac{1-x^2}{2}}}^{\sqrt{\frac{1-x^2}{2}}} 1 \cdot \sqrt{2}\, dy\, dx$$

$$= \sqrt{2} \int_{-1}^{1} (2) \sqrt{\frac{1-x^2}{2}}\, dx$$

$$= 2 \int_{-1}^{1} \sqrt{1-x^2}\, dx$$

Set $x = \cos u \Rightarrow u : \pi \to 0$

$dx = (-\sin u)\, du$

$$\Rightarrow 2 \int_{0}^{\pi} \sin^2 u\, du$$

$$= 2 \int_{0}^{\pi} \left(\frac{1-\cos 2u}{2}\right) du = \pi$$

16-5

5. $G(x,y) = \langle x, y, 1-3x+4y \rangle$

$$G_x \times G_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -3 \\ 0 & 1 & 4 \end{vmatrix}$$

$$= \langle 3, -4, 1 \rangle$$

$$\iint_S F \cdot (G_x \times G_y)\, dx\, dy$$

$$= \int_0^1 \int_0^1 \left(3y - 4(1-3x+4y) + x\right) dx\, dy$$

$$= -4$$

9. $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 9 - r^2 \end{cases} \Rightarrow G(r, \theta) = (r\cos\theta,\ r\sin\theta,\ 9 - r^2)$

$\cancel{r}\ \ 9 - r^2 \geq 0 \Rightarrow |r| \leq 3$

$\Rightarrow \begin{cases} 0 \leq \theta \leq \frac{\pi}{2} \\ 0 \leq r \leq 3 \end{cases}$

$$G_r \times G_\theta = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & -2r \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$$= (2r^2\cos\theta)\,i + (2r^2\sin\theta)\,j + r\,k$$

$$\Rightarrow \langle 2r^2\cos\theta,\ 2r^2\sin\theta,\ r \rangle$$

$$\int_0^3 \int_0^{\frac{\pi}{2}} \langle 9-r^2, 9-r^2, r\cos\theta \rangle \cdot \langle 2r^2\cos\theta, +2r^2\sin\theta, r \rangle \, d\theta \, dr$$

$$= \int_0^3 \int_0^{\frac{\pi}{2}} \big(18r^2\cos\theta - 2r^4\cos\theta + 18r^2\sin\theta - 2r^4\sin\theta + r^2\cos\theta \big) \, d\theta \, dr \qquad \cos\theta \Big|_{\frac{\pi}{2}}^{0}$$

$$= \int_0^3 \big(18r^2 - 2r^4 + 18r^2 - 2r^4 + r^2 \big) \, dr$$

$$= 37 \cdot \frac{r^3}{3} - \frac{4r^5}{5} \Big|_0^3$$

$$= \frac{693}{5}$$

17.3

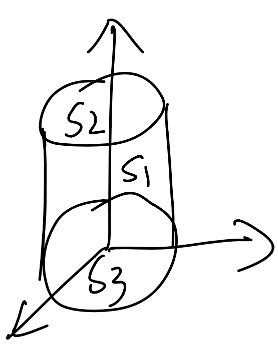
3. $\iiint_V \nabla \cdot F \, dV$

$$= \iint_{x^2+y^2 \le 1} \int_0^2 (2+0+0)\, dz\, dx\, dy$$

$$= 4 \iint_{x^2+y^2 \le 1} (1)\, dx\, dy = 4\pi$$

For $\iint_S F \cdot \vec{n} \, dS$

For S_1:



$$\begin{cases} G(\theta, z) = (\cos\theta, \sin\theta, z) \\ 0 \le \theta \le 2\pi \\ 0 \le z \le 2 \end{cases}$$

$$G_\theta \times G_z = \begin{vmatrix} i & j & k \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\cos\theta)\, i + (\sin\theta)\, j$$

$$\Rightarrow (\cos\theta, \sin\theta, 0)$$

$$\iint_{S_1} (2\cos\theta, 3z, 3\sin\theta) \cdot (\cos\theta, \sin\theta, 0)\, d\theta\, dz$$

$$= \int_0^{2\pi} \int_0^{2} (2\cos^2\theta + 3z\sin\theta)\, dz\, d\theta$$

$$= \int_0^{2\pi} \left(2\cos^2\theta\, z + 3\sin\theta \cdot \frac{z^2}{2} \Big|_0^2 \right) d\theta$$

$$= \int_0^{2\pi} (4\cos^2\theta + 6\sin\theta)\, d\theta$$

$$= \int_0^{2\pi} (\cos 2\theta + 1)\, 2\, d\theta$$

$$= 4\pi$$

For S_2: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 2 \end{cases}$

$$G(r, \theta) = (r\cos\theta,\ r\sin\theta,\ 1)$$

$$G_r \times G_\theta = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$$= r k \Rightarrow (0, 0, r)$$

So the integral is:

$$\int_0^1 \int_0^{2\pi} 3r^2\sin\theta \quad d\theta\, dr$$

$$= 0$$

The same value for S_3 i.e

$$G(r,\theta) = (r\cos\theta, r\sin\theta, 0)$$

$$G_r \times G_\theta = (0, 0, r)$$

The integral for S_3 is

$$\int_0^1 \int_0^{2\pi} 3r^2\sin\theta\, d\theta\, dr = 0$$

So $\iint_S \vec{F} \cdot \vec{n}\, dS = 4\pi.$