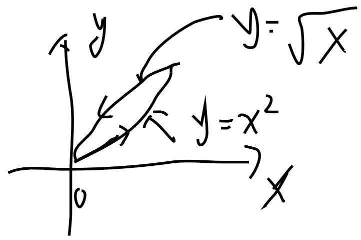
Hw 14 Solution

11.



$$C: \begin{cases} y = x^2, & 0 \le x \le 1 \quad \boxed{C_1} \\ y = \sqrt{x} & 1 \le x \le 0 \quad \boxed{C_2} \end{cases}$$

$$A = \oint_C -y\,dx$$

$$= \int_{C_1} -y\,dx + \int_{C_2} -y\,dx$$

C_1: parametrization $\vec{r}_1 = (t, t^2)$ $\quad \vec{r}_1{}' = (1, 2t) \quad 0 \le t \le 1$

$$\int_{C_1} (-y, 0)\cdot(1, 2t)\,dt = \int_0^1 (-t^2, 0)\cdot(1, 2t)\,dt$$

$$= \int_0^1 -t^2\,dt = -\frac{1}{3}$$

C_2. parametrization $\vec{r}_2 = (t, \sqrt{t})$ $\quad \vec{r}_2{}' = (1, \frac{1}{2}\frac{1}{\sqrt{t}}) \quad 1 \le t \le 0$

$$\int_{C_2} (-y, 0)\,\vec{r}_2{}'\,dt = \int_1^0 (-\sqrt{t}, 0)\cdot(1, \frac{1}{2}\frac{1}{\sqrt{t}})\,dt$$

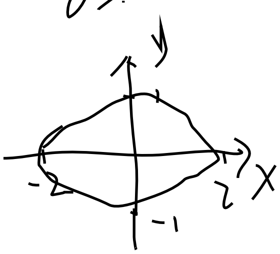
$$= \int_1^0 -\sqrt{t}\,dt = \frac{2}{3}$$

$$A = \int_{C_1} (-y)\,dx + \int_{C_2} (-y)\,dx = -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$$

19. By Stoke's Theorem

$$\iint_S \text{curl}\,\vec{F}\, dS = \oint_{\partial S} \vec{F} \cdot dr$$

∂S



$$\frac{x^2}{4} + y^2 = 1 \qquad x = 2\cos\theta \qquad 0 \le \theta \le 2\pi$$

$$y = \sin\theta \qquad z = 0$$

$$\vec{r} = (2\cos\theta, \sin\theta, 0) \qquad \vec{r}\,' = (-2\sin\theta, \cos\theta, 0)$$

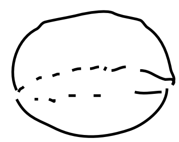
$$\oint_{\partial S} \vec{F} \cdot dr = \int_0^{2\pi} (z^2, x+z, y^2) \cdot (-2\sin\theta, \cos\theta, 0)\, d\theta$$

$$= \int_0^{2\pi} (0, 2\cos\theta, \sin^2\theta) \cdot (-2\sin\theta, \cos\theta, 0)\, d\theta$$

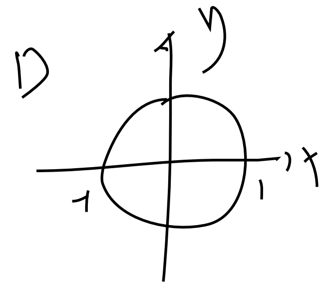
$$= \int_0^{2\pi} 2\cos^2\theta\, d\theta = \int_0^{2\pi} \cos 2\theta + 1\, d\theta$$

$$= 2\pi$$

33.



$x^2+y^2\le 1$



a) for D $\quad N=(0,0,-1)$

$$\vec{F}\cdot N=(x^2+y^2,0,z^2)\cdot(0,0,-1)=-z^2$$

on D, $z=0$ therefore $\displaystyle\iint_D \vec{F}\, N\, dA=0$

b) By the Divergence theorem.

$$\iiint_{\partial W}\vec{F}\cdot ds=\iiint_W \operatorname{div}\vec{F}\, dV$$

$$\iiint_{\partial W}\vec{F}\cdot ds=\iint_S \vec{F}\cdot ds+\underbrace{\iint_D \vec{F}\cdot ds}_{=0}=\iint_S \vec{F}\cdot ds$$

S parametrized $\quad x=\sin\varphi\cos\theta,\ y=\sin\varphi\sin\theta,\ z=\cos\varphi \quad \begin{matrix}0\le\varphi\le\frac{\pi}{2}\\ 0\le\theta\le 2\pi\end{matrix}$

$$N_s=\langle \sin^2\varphi\cos\theta,\ \sin^2\varphi\sin\theta,\ \sin\varphi\cos\varphi\rangle$$

$$\iint_S \vec{F}\, ds=\iint_S (x^2+y^2,0,z^2)\cdot N_s\, ds=\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\sin^4\varphi\cos\theta+\sin\varphi\cos^3\varphi\, d\theta\, d\varphi$$

$$=\int_0^{\frac{\pi}{2}} 0+2\pi\sin\varphi\cos^3\varphi\, d\varphi=\frac{\pi}{2}$$