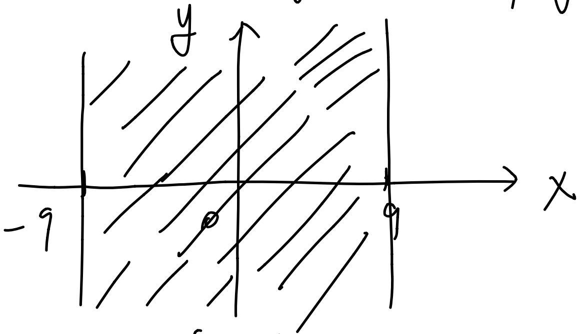
14.1

8. solution: Domain $= \{(x,y) : |x| \le 9, y \in \mathbb{R}\}$



14.2. 12. $\lim_{(x,y)\to(2,5)} \frac{f(x,y)}{f(x,y)+g(x,y)}$

$$= \frac{\lim_{(x,y)\to(2,5)} f(x,y)}{\lim_{(x,y)\to(2,5)} f(x,y) + \lim_{(x,y)\to(2,5)} g(x,y)}$$

$$= \frac{3}{3+7} = \frac{3}{10}$$

15. Solution: $\lim_{(x,y)\to(0,0)} f(x,y)$

if $y=mx$ then $\lim_{(x,y)\to(0,0)} f(x,y)$

$$= \lim_{(x,y)\to(0,0)} \frac{x^3+(mx)^3}{x(mx)^2}$$

$$= \lim_{(x,y)\to(0,0)} \frac{x^3+m^3x^3}{m^2x^3} = \frac{1+m^3}{m^2}$$

The limit depends on the value of m, therefore it doesn't exist.

18. Solution: Let $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(r,\theta)\to(0^+,??)} \frac{(r\cos\theta)^3}{(r\cos\theta)^2 + (r\sin\theta)^2}$$

$$= \lim_{(r,\theta)\to(0^+,??)} r\cos^3\theta$$

Observe that $\cos\theta$ is in $[-1,1]$ for all real θ.

$$0 = \lim_{r\to 0^+} -r \le \lim_{(r,\theta)\to(0^+,??)} r\cos^3\theta \le \lim_{r\to 0^+} r = 0$$

then the limit equals 0 by Squeeze theorem.

$$\lim_{(x,y)\to(0,0)} g(x,y) = \lim_{(r,\theta)\to(0^+,??)} \frac{(r\cos\theta)^2}{(r\cos\theta)^2 + (r\sin\theta)^2}$$

$$= \lim_{(r,\theta)\to(0^+,??)} \cos^2\theta$$

The limit depends on θ, therefore it doesn't exist.

21. Solution: Let $y = mx$, then.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = \lim_{(x,y)\to(0,0)} \frac{x(mx)}{3x^2+2(mx)^2}$$

$$= \lim_{(x,y)\to(0,0)} \frac{mx^2}{3x^2+2m^2x^2} = \frac{m}{3+2m^2}$$

The limit depends on the value of m, it doesn't exist.

32. Solution: Let $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ then.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{(r,\theta)\to(0^+,??)} \frac{r^2\cos\theta\sin\theta}{\sqrt{r^2\cos^2\theta + r^2\sin^2\theta}}$$

$$= \lim_{(r,\theta)\to(0^+,??)} r\,\frac{\sin 2\theta}{2}$$

Observe that $\sin 2\theta \in [-1,1]$ for all θ in R.
then,

$$0 = \lim_{r\to 0^+} \frac{-r}{2} \le \lim_{(r,\theta)\to(0^+,??)} r\,\frac{\sin 2\theta}{2} \le \lim_{r\to 0^+} \frac{r}{2} = 0$$

the limit is 0 by squeeze theorem.

42. solution: Let $\begin{cases} u = x-1 \\ v = y-1 \end{cases}$ then

$$\lim_{(x,y)\to(1,1)} \frac{x^2+y^2-2}{|x-1|+|y-1|} = \lim_{(u,v)\to(0,0)} \frac{(u+1)^2+(v+1)^2-2}{|u|+|v|}$$

$$= \lim_{(u,v)\to(0,0)} \frac{u^2+2u+v^2+2v}{|u|+|v|}$$

choosing the direction $u=0$ $v\to 0$
then the limit changes into

$$\lim_{v\to 0} \frac{v^2+2v}{|v|}$$

Since $\displaystyle\lim_{v\to 0^+} \frac{v^2+2v}{|v|} = \lim_{v\to 0^+} \frac{v^2+2v}{v}$

$\displaystyle = \lim_{v\to 0^+} v+2 = 2$

$\displaystyle\lim_{v\to 0^-} \frac{v^2+2v}{|v|} = \lim_{v\to 0^-} \frac{v^2+2v}{-v}$

$\displaystyle = \lim_{v\to 0^-} -v-2$

$= -2$

$2 \neq -2$, $\displaystyle\lim_{v\to 0} \frac{v^2+2v}{|v|}$ DNE $\Rightarrow \displaystyle\lim_{(x,y)\to(1,1)} \frac{x^2+y-2}{|x-1|+|y-1|}$ DNE.

47. Solution: Yes.

Let $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ then

$$f(x,y) = f(r) = \begin{cases} r^2 & r^2<1 \\ 1 & r^2 \geq 1 \end{cases}$$

$$\lim_{r\to 1^+} f(r) = \lim_{r\to 1^+} 1 = 1$$

$$\lim_{r\to 1^-} f(r) = \lim_{r\to 1^-} r^2 = 1^2 = 1$$

then $\lim_{r\to 1} f(r) = 1$

also since $f(1) = 1$, then

$$\lim_{r\to 1} f(r) = f(1)$$

$f(r)$ is continuous at $r=1$

$\Rightarrow f(x,y)$ is continuous on R^2.