HW 2 Solution

section 14.3 :

9. slope of tangent line of the curve at (1,1,6)

$$\frac{\partial f}{\partial x}\Big|_{(1,1)} = \frac{\partial}{\partial x}(x^4+6xy-y^4)\Big|_{(1,1)}$$
$$= 4x^3+6y\Big|_{(1,1)} = 4(1)^3+6 = 10$$

19. $z=\frac{x}{y}$:

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{y}\right) = \frac{1}{y}$$
$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x}{y}\right) = \frac{-x}{y^2}$$

25. $z=\cos\left(\frac{1-x}{y}\right)$:

$$\frac{\partial z}{\partial x} = \left(-\sin\left(\frac{1-x}{y}\right)\right)\cdot\frac{\partial}{\partial x}\left(\frac{1-x}{y}\right)$$
$$= \left(-\sin\left(\frac{1-x}{y}\right)\right)\left(-\frac{1}{y}\right) = \frac{1}{y}\sin\left(\frac{1-x}{y}\right)$$
$$\frac{\partial z}{\partial y} = \left(-\sin\left(\frac{1-x}{y}\right)\right)\frac{\partial}{\partial y}\left(\frac{1-x}{y}\right)$$
$$= \left(-\sin\left(\frac{1-x}{y}\right)\right)(-1)\frac{(1-x)}{y^2}$$
$$= \frac{(1-x)}{y^2}\sin\left(\frac{1-x}{y}\right)$$

43. $f_x(x,y) = \frac{\partial f}{\partial x}\Big|_{(1,2)} = \frac{\partial}{\partial x}(3x^2y+4x^3y^2-7xy^5)\Big|_{(1,2)}$

$$= 6xy+12x^2y^2-7y^5\Big|_{(1,2)}$$
$$= 12+48-7(2^5) = -164$$

39. $w = xy^2z^3$:

$$\frac{\partial w}{\partial x} = y^2z^3 \quad \frac{\partial w}{\partial y} = 2xyz^3$$
$$\frac{\partial w}{\partial z} = 3xy^2z^2$$

49: (a) The level curve is more intense at p than Q,

With same Δf vertically (where $\Delta f>0$), $0<\Delta y_p<\Delta y_Q$

Since $f_y \approx \frac{\Delta f}{\Delta y}$ then $f_y(p) > f_y(Q)$.
also with same Δf horizontally (where $\Delta f < 0$), $0 < \Delta x_p < \Delta x_Q$
Since $f_x \approx \frac{\Delta f}{\Delta x}$ then $f_x(p) < f_x(Q) \Rightarrow f_x$ is more negative

(b) For fixed $x=a$, $f_x(a,y) \approx \frac{\Delta f}{\Delta x} = \frac{f(a+\Delta x, y) - f(a,y)}{\Delta x}$

The curves are more and more sparse with increasing of y, which means with same Δf (where $\Delta f<0$), Δx is getting larger, then the absolute value of $\frac{\Delta f}{\Delta x}$ is getting smaller, thus $|f_x(a,y)|$ is decreasing with y.

53. $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(3x^2y - 6xy^4)\right)$

$= \frac{\partial}{\partial x}(6xy - 6y^4)$

$= 6y$

$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(3x^2y - 6xy^4)\right)$

$= \frac{\partial}{\partial y}(3x^2 - 24xy^3)$

$= -72xy^2$

57. $f_{yy}(2,3) = \frac{\partial^2 f}{\partial y^2}\Big|_{(2,3)} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(x \ln y^2)\right)\Big|_{(2,3)}$

$= \frac{\partial}{\partial y}\left(2x\frac{1}{y}\right)\Big|_{(2,3)}$

$= \frac{-2x}{y^2}\Big|_{(2,3)} = \frac{-4}{9}$

61. $f_u = \frac{\partial}{\partial u}(\cos(u+v^2)) = -\sin(u+v^2)$

$f_{uu} = \frac{\partial}{\partial u}(-\sin(u+v^2)) = -\cos(u+v^2)$

$f_{uuv} = \frac{\partial}{\partial v}(-\cos(u+v^2)) = \sin(u+v^2) \cdot \frac{\partial}{\partial v}(u+v^2)$

$= \sin(u+v^2)(2v)$

75. (b). $u_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(e^x \cos y)\right)$

$= \frac{\partial}{\partial x}((\cos y) e^x)$

$= (\cos y) e^x$

$u_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(e^x \cos y)\right)$

$= \frac{\partial}{\partial y}(e^x(-\sin y))$

$= e^x(-\cos y)$

So, $u_{xx} + u_{yy} = e^x \cos y + e^x(-\cos y)$

$= 0$

$\Rightarrow$ u is a harmonic function.