HW 4 solution

Section 14.5

34. $D_{\vec{u}} f(p) = \|\nabla f\| \cos\theta$

$\Rightarrow f$ has maxium rate of change along the gradient direction.

$$\nabla f(p) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)\Big|_p = (2x-y,\ 2y-x)\Big|_{(-1,4)}$$
$$= (-6, 9)$$

Rate of change : $\|\nabla f\| = \|(-6,9)\| = \sqrt{117}$

37. $D_{\frac{\vec{v}}{\|\vec{v}\|}} f = \nabla f_p \cdot \frac{\vec{v}}{\|\vec{v}\|}$

$$= (2,-4,4) \cdot \frac{(2,1,3)}{\sqrt{14}}$$
$$= \frac{12}{\sqrt{14}}$$

Since $D_{\frac{\vec{v}}{\|\vec{v}\|}} f > 0$, f is increasing at p in the $\vec{v}$ direction.

38 (a). $\nabla f_p = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)\Big|_p$

$$= \left((1+2x^2)e^{x^2-y},\ -xe^{x^2-y}\right)\Big|_{(1,1)}$$
$$= (3, -1)$$

$$\Rightarrow \|\nabla f_p\| = \sqrt{3^2+(-1)^2} = \sqrt{10}$$

(b) Rate of change:

$$D_{\frac{\nabla f}{\|\nabla f\|}} f(p) = \|\nabla f\| \cdot \left\|\frac{\nabla f}{\|\nabla f\|}\right\|$$

$$= \|\nabla f\| = \sqrt{10}$$

(c). assume u is vector as required

then rate of change along u is:

$$D_{\frac{\vec{u}}{\|\vec{u}\|}} f = \|\nabla f\| \cdot \left\|\frac{\vec{u}}{\|\vec{u}\|}\right\| \cdot \cos(45^\circ)$$

$$= \sqrt{10} \cdot 1 \cdot \frac{\sqrt{2}}{2}$$

$$= \sqrt{5}$$

47. $F(x, y, z) = xz + 2x^2y + y^2z^3$

$$\nabla F(p) = (z + 4xy,\ 2x^2 + 2yz^3,\ x + 3y^2z^2)\big|_{(2,1,1)}$$

$$= (9, 10, 5)$$

$\Rightarrow$ The tangent plane at p:

$$9(x-2) + 10(y-1) + 5(z-1) = 0$$

$$\Rightarrow 9x + 10y + 5z = 33$$

Section 14.6

6. $\frac{\partial R}{\partial v} = \frac{\partial R}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial R}{\partial y}\cdot\frac{\partial y}{\partial v}$

$= 3(x-2y)^2(0) + 3(x-2y)^2(-2)(w)v^{w-1}$

$= -6(x-2y)^2(w)v^{w-1}$

$= -6(w^2-2v^w)^2(w)v^{w-1}$

$\frac{\partial R}{\partial w} = \frac{\partial R}{\partial x}\cdot\frac{\partial x}{\partial w} + \frac{\partial R}{\partial y}\cdot\frac{\partial y}{\partial w}$

$= 3(x-2y)^2(2w) + 3(x-2y)^2(-2)v^w \ln v$

$= 6(x-2y)^2(w - v^w \ln v)$

$= 6(w^2-2v^w)^2(w - v^w \ln v)$

15. $\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial u}$

$= (2x)(e^u\cos(v)) + (-2y)(e^u\sin(v))$

$= 2e^u(x\cos(v) - y\sin(v))$

$= 2e^u(e^u(\cos(v))^2 - e^u(\sin v)^2)$

$= 2e^{2u}\cos(2v)$

$\frac{\partial g}{\partial u}\Big|_{(0,1)} = 2e^{2u}\cos(2v)\Big|_{(0,1)}$

$= 2\cos(2)$

43. Let $v = x - ct$

$u(x,t) = f(v(x,t))$

$$\frac{\partial u}{\partial t} = f'(v(x,t)) \cdot \frac{\partial v}{\partial t} = f'(x-ct)(-c)$$

$$\frac{\partial u}{\partial x} = f'(v(x,t)) \cdot \frac{\partial v}{\partial x} = f'(x-ct)(1)$$

$$\Rightarrow \frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = f'(x-ct)(-c) + c\, f'(x-ct)$$

$$= 0$$

31. $\frac{\partial}{\partial y}\left(e^{xy} + \sin(xz) + y\right) = 0$

$$\Rightarrow e^{xy}(x) + \cos(xz)(x\, z_y) + 1 = 0$$

$$\Rightarrow x\cos(xz)\, z_y = -1 - xe^{xy}$$

$$z_y = \frac{-1 - xe^{xy}}{x\cos(xz)}$$