HW 6.

section 15.1

3. (A) $S_{3,2} = (1)(1)\big(f(1.5,1.5) + f(2.5,1.5) + f(3.5,1.5)$
$+ f(1.5,2.5) + f(2.5,2.5) + f(3.5,2.5)\big)$

$= (1)\big(2(1.5)+1.5 + 2(2.5)+1.5 + 2(3.5)+1.5$
$+2(1.5)+2.5 + 2(2.5)+2.5 + 2(3.5)+2.5\big)$

$= 42$

(B) $S_{3,2} = (1)(1)\big(f(1.5,1.5) + f(2,1) + f(3.5,1.5)$
$+ f(2,3) + f(2.5,2.5) + f(4,3)\big)$

$= 43.5$

23. $\int_{-1}^{1}\int_{0}^{\pi} x^2 \sin y \, dy \, dx$

$= \int_{-1}^{1} x^2 \int_{0}^{\pi} \sin y \, dy \, dx$

$= \int_{-1}^{1} x^2 \left(-\cos y \Big|_0^{\pi}\right) dx$

$= \int_{-1}^{1} x^2 (2) \, dx$

$= 2 \int_{-1}^{1} x^2 dx$

$= \frac{2}{3} x^3 \Big|_{-1}^{1} = \frac{4}{3}$

41. $\iint_{[0,2]\times[0,\frac{\pi}{4}]} e^x \sin y \, dA$

$= \int_0^2 \int_0^{\frac{\pi}{4}} e^x \sin y \, dy \, dx$

$= \int_0^2 e^x \int_0^{\frac{\pi}{4}} \sin y \, dy \, dx$

$= \int_0^2 e^x \left(-\cos y \Big|_0^{\frac{\pi}{4}}\right) dx$

$= \int_0^2 e^x \left(1-\frac{\sqrt{2}}{2}\right) dx$

$= \left(1-\frac{\sqrt{2}}{2}\right) \int_0^2 e^x \, dx = \left(1-\frac{\sqrt{2}}{2}\right)(e^2-1)$

49. (a) with x is easier because

$$\frac{d \ln(1+xy)}{dx} = \frac{1}{1+xy}(y) = \frac{y}{1+xy}$$

then $d \ln(1+xy) = \frac{y}{1+xy} dx$

$\Rightarrow \int \frac{y}{1+xy} dx = \int d \ln(1+xy) = \ln(1+xy) + C$

For integrating with respect to y, It takes more steps.

(b).

$$\iint_{[0,1]\times[0,1]} \frac{y}{1+xy}\, dy\, dx$$

$$= \int_0^1 \int_0^1 \frac{y}{1+xy}\, dx\, dy$$

Fubini theorem we can change order since $f(x,y)$ is continuous on R.

$$\overset{\text{by (a)}}{=} \int_0^1 \left(\ln(1+xy) \Big|_0^1 \right) dy$$

$$= \int_0^1 \left(\ln(1+y) - \ln(1+0) \right) dy$$

$$= \int_0^1 \ln(1+y)\, dy$$

do substitution set $1+y = u$

then $y: 0 \to 1$

$u = 1+y : 1 \to 2$

$$\int_0^1 \ln(1+y)\, dy = \int_1^2 \ln(u)\, du$$

$$= (\ln(u))(u) \Big|_1^2 - \int_1^2 u\, d\ln u$$

$$= (2\ln 2 - (1)\ln 1) - \int_1^2 u \cdot \left(\frac{1}{u}\right) du$$

$$= 2\ln 2 - \int_1^2 (1)\, du$$

$$= 2\ln 2 - \left(u \Big|_1^2 \right) = 2\ln 2 - (2-1)$$

$$= 2\ln 2 - 1$$