HW 8.

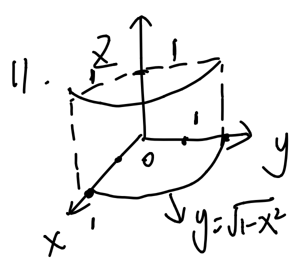
9. $$\int_0^1 \int_0^x \int_y^x (x+y)\,dz\,dy\,dx$$

$$= \int_0^1 \int_0^x \left((x+y)z \Big|_y^x \right) dy\,dx$$

$$= \int_0^1 \int_0^x (x^2 - y^2)\,dy\,dx$$

$$= \int_0^1 \left(x^2 y - \frac{y^3}{3} \Big|_0^x \right) dx$$

$$= \int_0^1 \left(\frac{2}{3} x^3 \right) dx = \frac{2}{3}\left(\frac{1}{4}\right) x^4 \Big|_0^1 = \frac{1}{6}$$

11. 

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^1 (xyz)\,dz\,dy\,dx$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \left((xy)\frac{z^2}{2} \Big|_0^1 \right) dy\,dx$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \left(\frac{xy}{2} \right) dy\,dx$$

$$= \int_0^1 \left(\frac{x}{2}\right) \cdot \left(\frac{y^2}{2} \Big|_0^{\sqrt{1-x^2}} \right) dx$$

$$= \int_0^1 \left(\frac{x}{2}\right)\left(\frac{1-x^2}{2}\right) dx$$

$$= \frac{1}{4} \int_0^1 (x - x^3)\,dx$$

$$= \frac{1}{4} \left(\frac{x^2}{2} - \frac{x^4}{4} \Big|_0^1 \right) = \frac{1}{16}$$

15. $\int_0^1 \int_0^x \int_0^{\sqrt{9-x^2-y^2}} z \, dz \, dy \, dx$

$=$

$= \int_0^1 \int_0^x \left(\frac{z^2}{2} \Big|_0^{\sqrt{9-x^2-y^2}} \right) dy \, dx$

$= \int_0^1 \int_0^x \left(\frac{9-x^2-y^2}{2} \right) dy \, dx$

$= \int_0^1 \left(\left(\frac{9-x^2}{2}\right) y - \frac{y^3}{6} \right) \Big|_0^x \, dx$

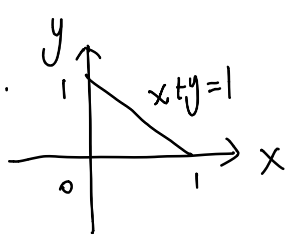
$= \int_0^1 \left(\left(\frac{9-x^2}{2}\right) x - \frac{x^3}{6} \right) dx$

$= \int_0^1 \left(\frac{9}{2} x - \frac{x^3}{2} - \frac{x^3}{6} \right) dx$

$= \frac{9}{4} x^2 - \frac{x^4}{8} - \frac{x^4}{24} \Big|_0^1$

$= \frac{9}{4} - \frac{1}{8} - \frac{1}{24} = \frac{50}{24} = \frac{25}{12}$

26.



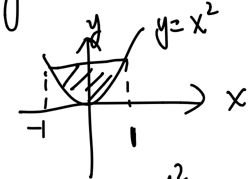
$\int_0^1 \int_0^{1-x} \int_{1-x-y}^{x^2+y^2+2} f(x,y,z) \, dz \, dy \, dx$

33. at the intersection points:

$$\begin{cases} z = 1 - y^2 \\ y = x^2 \end{cases} \Rightarrow z = 1 - x^4 \quad z \geq 0 \Rightarrow x^4 \leq 1$$
$$\Rightarrow |x| \leq 1$$
$$\Rightarrow -1 \leq x \leq 1$$

Height: $0 \to 1 - y^2$

D:



$$V = \int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y^2} (1)\, dz\, dy\, dx$$

$$= \int_{-1}^{1} \int_{x^2}^{1} \left(z \Big|_{0}^{1-y^2} \right) dy\, dx$$

$$= \int_{-1}^{1} \int_{x^2}^{1} (1 - y^2)\, dy\, dx$$

$$= \int_{-1}^{1} \left(y - \frac{y^3}{3} \Big|_{x^2}^{1} \right) dx$$

$$= \int_{-1}^{1} \left(\frac{2}{3} - \left(x^2 - \frac{x^6}{3} \right) \right) dx$$

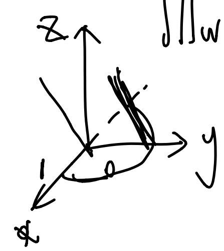
$$= \int_{-1}^{1} \left(\frac{2}{3} - x^2 + \frac{x^6}{3} \right) dx$$

$$= \left(\frac{2}{3}x - \frac{x^3}{3} + \frac{x^7}{21} \right) \Big|_{-1}^{1}$$

$$= \left(\frac{2}{3} - \frac{1}{3} + \frac{1}{21} \right) - \left(\frac{2}{-3} + \frac{1}{3} - \frac{1}{21} \right)$$

$$= \frac{4}{3} - \frac{2}{3} + \frac{2}{21} = \frac{16}{21}$$

41. $\bar{f} = \dfrac{\iiint_W e^y \, dV}{\iiint_W 1 \, dV}$



$\iiint_W e^y dV$

$= \int_0^1 \int_0^{1-x^2} \int_0^x e^y \, dz \, dy \, dx$

$= \int_0^1 \int_0^{1-x^2} \left(e^y z \Big|_0^x \right) dy \, dx$

$= \int_0^1 \int_0^{1-x^2} (e^y x) \, dy \, dx$

$= \int_0^1 x \left(e^y \Big|_0^{1-x^2} \right) dx$

$= \int_0^1 x \left(e^{1-x^2} - 1 \right) dx$

$= \int_0^1 x e^{1-x^2} dx - \int_0^1 x \, dx$

$= \frac{1}{2} \left(-e^{1-x^2} \Big|_0^1 \right) - \frac{1}{2}$

$= \frac{1}{2} (e - 1) - \frac{1}{2}$

$= \frac{e}{2} - 1$

$\iiint_W 1 \, dV = \int_0^1 \int_0^{1-x^2} \int_0^x (1) \, dz \, dy \, dx$

$= \int_0^1 \int_0^{1-x^2} \left(z \Big|_0^x \right) dy \, dx$

$$= \int_0^1 \int_0^{1-x^2} (x)\, dy\, dx$$

$$= \int_0^1 x(1-x^2)\, dx$$

$$= \int_0^1 (x - x^3)\, dx = \frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1$$

$$= \frac{1}{4}$$

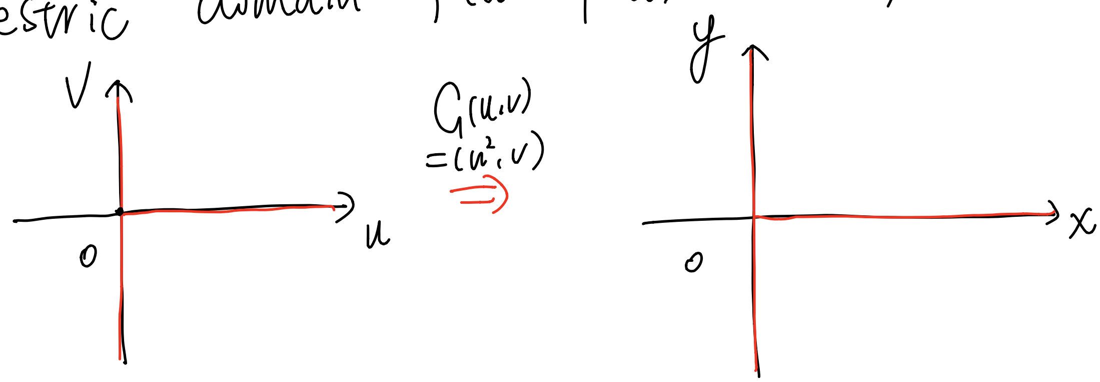
$$\Rightarrow \bar{f} = \frac{\frac{e}{2} - 1}{\frac{1}{4}} = 2e - 4$$

15.6

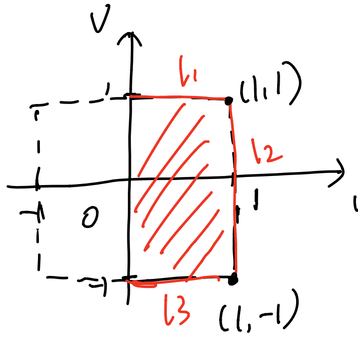
3. G is not one to one.

Restric domain $\{(u,v) \mid u \geq 0,\ v \in \mathbb{R}\}$

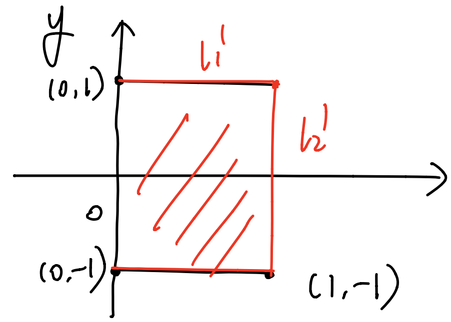
(a).



(b)



$G(u,v) = (u^2, v) \Rightarrow$



$$\begin{cases} x = u^2 \\ y = v \end{cases}$$

$l_1: \ v = 1 \quad 0 \le u \le 1$

$$\Rightarrow \begin{cases} 0 \le x \le 1 \\ y = 1 \end{cases} \quad l_1'$$

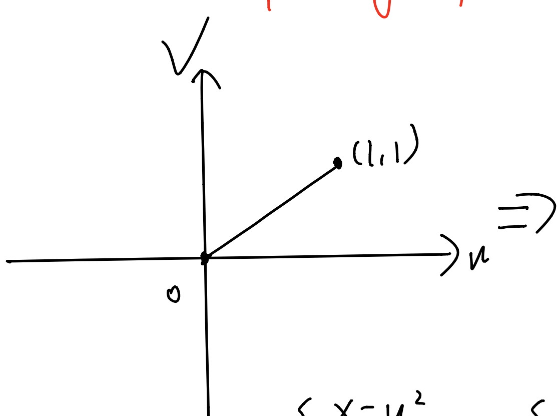
$l_2: \ u = 1 \quad -1 \le v \le 1$

$$\begin{cases} x = 1 \\ -1 \le y \le 1 \end{cases} \quad l_2'$$

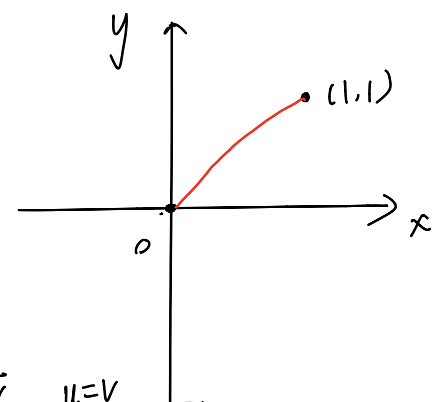
$l_3: \ v = -1 \quad 0 \le u \le 1$

$$\begin{cases} 0 \le x \le 1 \\ y = -1 \end{cases}$$

(c)



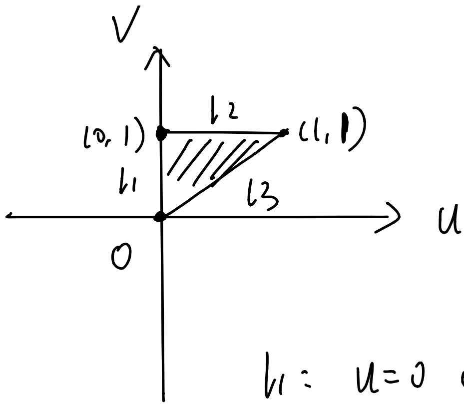
$\Rightarrow$



$$\begin{cases} x = u^2 \\ y = v \end{cases} \Rightarrow \begin{cases} u = \sqrt{x} \\ v = y \end{cases} \xRightarrow{u=v} \sqrt{x} = y$$

$$x = y^2$$

(d)



$$\begin{cases} x = u^2 \\ y = v \end{cases}$$

l_1: $u = 0 \quad 0 \le v \le 1$

$$\xrightarrow{G} l_1' \begin{cases} x = 0 \\ 0 \le y \le 1 \end{cases}$$

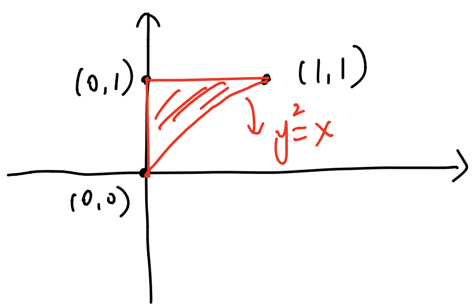
l_2: $v = 1 \quad 0 \le u \le 1$

$$\xrightarrow{G} l_2' \begin{cases} 0 \le x \le 1 \\ y = 1 \end{cases}$$

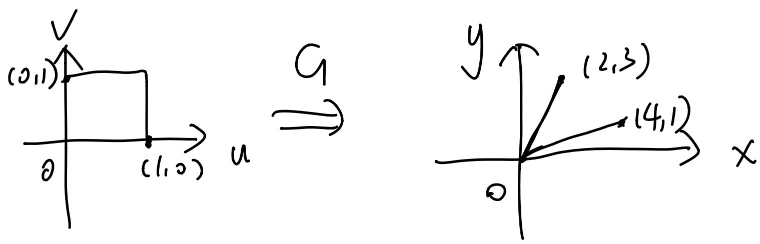
l_3: $0 \le u \le 1$, $v = u$

$$\xrightarrow{G} l_3' \begin{cases} 0 \le x \le 1 \\ y = \sqrt{x} \end{cases}$$

$l_1', l_2', l_3' \Rightarrow$



19.



$$G(0,1) = (2,3)$$

$$G(1,0) = (4,1)$$

Since G is linear map, we assume $G(u,v) = (au+bv,\ cu+dv)$

$$G(0,1) = (2,3) \Rightarrow \begin{cases} a(0)+b(1)=2 \\ c(0)+d(1)=3 \end{cases} \Rightarrow \begin{cases} b=2 \\ d=3 \end{cases}$$

$$G(1,0) = (4,1) \Rightarrow \begin{cases} a(1)+b(0)=4 \\ c(1)+d(0)=1 \end{cases} \Rightarrow \begin{cases} a=4 \\ c=1 \end{cases}$$

$$\Rightarrow G(u,v) = (4u+2v,\ u+3v)$$