Hw1 Solution

Section 1.3

1. Second order, linear.
3. Fourth order, linear.
4. Second order, nonlinear.
7. Solution:
   $y = 3t + t^2$, $y' = 3 + 2t$
   $ty' - y = t(3 + 2t) - (3t + t^2) = 3t + 2t^2 - 3t - t^2 = t^2$.
9. Solution:
   $y_1 = t^{-2}$, $y_1' = -2t^{-3}$, $y_1'' = 6t^{-4}$
   $t^2 y_1'' + 5ty_1' + 4y_1 = t^2(6t^{-4}) + 5t(-2t^{-3}) + 4t^{-2} = 6t^{-2} - 10t^{-2} + 4t^{-2} = 0$.

   $y_2 = t^{-2}\ln t$, $y_2' = -2t^{-3}\ln t + t^{-2}/t = -2t^{-3}\ln t + t^{-3}$,
   $y_2'' = 6t^{-4}\ln t - 2t^{-4} - 3t^{-4} = 6t^{-4}\ln t - 5t^{-4}$
   $t^2 y_2'' + 5ty_2' + 4y_2 = t^2(6t^{-4}\ln t - 5t^{-4}) + 5t(-2t^{-3}\ln t + t^{-3}) + 4t^{-2}\ln t = 6t^{-2}\ln t - 10t^{-2}\ln t + 4t^{-2}\ln t - 5t^{-2} + 5t^{-2} = 0$.

12. Solution:
    $y = e^{rt}$, $y' = re^{rt}$, $y'' = r^2 e^{rt}$
    $y'' + y' - 6y = r^2 e^{rt} + re^{rt} - 6e^{rt} = (r^2 + r - 6)e^{rt} = 0 \Longrightarrow r^2 + r - 6 = 0 \Longrightarrow r = 2$ or $r = -3$.

15. Solution:
    $y = t^r$, $y' = rt^{r-1}$, $y'' = r(r-1)t^{r-2}$
    $t^2 y'' - 4ty' + 4y = t^2 r(r-1)t^{r-2} - 4trt^{r-1} + 4t^r = (r(r-1) - 4r + 4)t^r = 0 \Longrightarrow r^2 - 5r + 4 = 0 \Longrightarrow r = 1$ or $r = 4$.

Section 2.1

3c. Solution:
    (1) $p(t) = 1$, $g(t) = te^{-t} + 1$
    (2) $\mu = e^{\int p(t)dt} = e^{\int 1dt} = e^t$
    (3) $\int \mu(t)g(t)dt = \int e^t\left(te^{-t} + 1\right)dt = t^2/2 + e^t + c$
    (4) $y(t) = \frac{\int \mu(t)g(t)dt}{\mu(t)} = \frac{t^2/2 + e^t + c}{e^t} == \frac{t^2}{2}e^{-t} + 1 + ce^{-t}$.
    When $t \to \infty$, $y(t) \to 1$

5c. Solution:

(1) $p(t) = -2,\ g(t) = 3e^t$

(2) $\mu = e^{\int p(t)dt} = e^{\int -2dt} = e^{-2t}$

(3) $\int \mu(t)g(t)dt = \int e^{-2t}3e^t dt = -3e^{-t} + c$

(4) $y(t) = \frac{\int \mu(t)g(t)dt}{\mu(t)} = \frac{-3e^{-t}+c}{e^{-2t}} = -3e^t + ce^{2t}$.

When $t \to \infty$, $y(t) \to +\infty$ if $c > 0$ or $y(t) \to -\infty$ if $c \leq 0$

6c. Solution:

(1) $p(t) = -\frac{1}{t},\ g(t) = te^{-t}$

(2) $\mu = e^{\int p(t)dt} = e^{\int -\frac{1}{t}dt} = \frac{1}{t}$

(3) $\int \mu(t)g(t)dt = \int \frac{1}{t}te^{-t}dt = -e^{-t} + c$

(4) $y(t) = \frac{\int \mu(t)g(t)dt}{\mu(t)} = \frac{-e^{-t}+c}{\frac{1}{t}} = -te^{-t} + ct$.

When $t \to \infty$, $y(t) \to +\infty$ if $c > 0$ or $y(t) \to -\infty$ if $c < 0$, $y(t) \to 0$ if $c = 0$