Hw10 Solution

Section 3.7 1. Solution:

$$\begin{aligned} u &= 3\cos 2t + 4\sin 2t \\ &= \sqrt{3^2+4^2}\cos(2t-\delta) \\ &= 5\cos(2t-\delta), \end{aligned}$$

where

$$\tan\delta = \frac{4}{3} \Longrightarrow \delta \approx 0.9273.$$

Therefore, $\omega_0 = 2$, $R = 5$, and $\delta \approx 0.9273$.

2. Solution:

$$\begin{aligned} u &= -2\cos \pi t - 3\sin \pi t \\ &= \sqrt{(-2)^2+(-3)^2}\cos(\pi t-\delta) \\ &= \sqrt{13}\cos(\pi t-\delta), \end{aligned}$$

where

$$\tan\delta = \pi + \tan\left(\frac{-3}{-2}\right) \Longrightarrow \delta \approx 4.1244.$$

Therefore, $\omega_0 = \pi$, $R = \sqrt{13}$, and $\delta \approx 4.1244$.

4. Solution:

$w = 3,\ mg = w \Longrightarrow m = \frac{w}{g} = \frac{3}{32}$.

$kL = mg \Longrightarrow k = \frac{mg}{L} = \frac{3}{3/12} = 12$.

We have $mu'' + ku = 0 \Longrightarrow \frac{3}{32}u'' + 12u = 0$ with $u(0) = -1/12$ (the mass pushed upward 1 inch) and $u'(0) = 2$ (downward velocity 2ft/s).

We solve this initial value problem as follows:

$2/32r^2 + 12 = 0 \Longrightarrow r = \frac{\pm\sqrt{-4\cdot 12\cdot 2/32}}{2\cdot 3/32} = \pm 8\sqrt{2}i$.

We have $\lambda = 0, \quad \mu = 8\sqrt{2}$ and

$$u_1(t) = e^{\lambda t}\cos\mu t = \cos 8\sqrt{2}t \quad u_2(t) = e^{\lambda t}\sin\mu t = \sin 8\sqrt{2}t.$$

The general solution is $\boxed{u = C_1u_1 + C_2u_2 = C_1\cos 8\sqrt{2}t + C_2\sin 8\sqrt{2}t.}$

$u' = -8\sqrt{2}C_1 \sin 8\sqrt{2}t + 8\sqrt{2}C_2 \cos 8\sqrt{2}t.$

$$\begin{aligned} u(0) = -1/12 &\implies C_1 = -1/12 \\ u'(0) = 2 &\implies 8\sqrt{2}2C_2 = 2 \\ &\implies \\ & \quad C_1 = -1/12 \quad C_2 = \sqrt{2}/8 \end{aligned}$$

Therefore the solution is $\boxed{u(t) = -1/12 \cos 8\sqrt{2}t + \sqrt{2}/8 \sin 8\sqrt{2}t.}$

$\omega = 8\sqrt{2}$, $T = \frac{2\pi}{\omega} = \sqrt{2}\pi/8$, $R = \sqrt{(-1/12)^2 + (\sqrt{2}/8)^2} = \sqrt{\frac{11}{288}} \approx 0.1954$,
$\tan\delta = \sqrt{2}/8/(-1/12) = -3\sqrt{2}/2 \to \delta = \pi - \arctan(3\sqrt{2}/2) \approx 2.0113.$

Section 7.1 4. $u'' + 0.25u' + 4u = 2\cos(3t), \quad u(0) = 1, u'(0) = -2$

Solution: Let $x_1 = u$, $x_2 = x_1' = u'$ and we have $x_1' = x_2$.

$$u'' + 0.25u' + 4u = x_2' + 0.25x_2 + 4x_1 = 2\cos(3t) \implies x_2' = -0.25x_2 - 4x_1 + 2\cos(3t).$$

Therefore

$$\boxed{\left\{ \begin{array}{lcl} x_1' &=& x_2 \\ x_2' &=& -0.25x_2 - 4x_1 + 2\cos(3t). \end{array} \right. \quad \text{with the initial condition } x_1(0) = 1 \text{ and } x_2(0) = -2.}$$

Section 7.2 4. $A = \begin{pmatrix} 3-2i & 1+i \\ 2-i & -2+3i \end{pmatrix}$

Solution:

a. $A^T = \begin{pmatrix} 3-2i & 2-i \\ 1+i & -2+3i \end{pmatrix}$

b. $\bar{A} = \begin{pmatrix} 3+2i & 1-i \\ 2+i & -2-3i \end{pmatrix}$

c. $A^* = \begin{pmatrix} 3+2i & 2+i \\ 1+i & -2+3i \end{pmatrix}$