Hw11

Section 7.3 6 Let $c_1x^{(1)} + c_2x^{(2)} + c_3x^{(3)} = 0$, then

$$\begin{bmatrix} c_1 \\ c_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \\ c_2 \end{bmatrix} + \begin{bmatrix} c_3 \\ 0 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

We have $\begin{cases} c_1 + c_3 &= 0 \\ c_1 + c_2 &= 0 \\ c_2 + c_3 &= 0. \end{cases}$

Therefore $c_1 = c_2 = c_3 = 0$ and $x^{(1)}$, $x^{(2)}$, and $x^{(3)}$ are linearly independent.

8 Let $c_1x^{(1)} + c_2x^{(2)} + c_3x^{(3)} + c_4x^{(4)} = 0$, then

$$\begin{bmatrix} c_1 \\ 2c_1 \\ -c_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2c_2 \\ 3c_2 \\ c_2 \\ -c_2 \end{bmatrix} + \begin{bmatrix} -c_3 \\ 0 \\ 2c_3 \\ 2c_3 \end{bmatrix} + \begin{bmatrix} 3c_4 \\ -c_4 \\ c_4 \\ 3c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

We have $\begin{cases} c_1 + 2c_2 - c_3 + 3c_4 &= 0 \\ 2c_1 + 3c_2 - c_4 &= 0 \\ -c_1 + c_2 + 2c_3 + c_4 &= 0 \\ -c_2 + 2c_3 + 3c_4 &= 0. \end{cases}$

Therefore $c_1 = c_2 = c_3 = c_4 = 0$ and $x^{(1)}$, $x^{(2)}$, $x^{(3}$, and $x^{(4)}$ are linearly independent.

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$$|A - \lambda I| = \begin{vmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{vmatrix} = (\lambda-5)(\lambda-1)+3 = \lambda^2-6\lambda+8 = 0 \to \lambda_1 = 2, \lambda_2 = 4.$$

For $\lambda_1 = 2$

$$\begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Longrightarrow 5x_1 - x_2 = 2x_1 \Longrightarrow x_2 = 3x_1.$$

The eigenvector is $\xi_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

For $\lambda_2 = 4$

$$\begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Longrightarrow 5x_1 - x_2 = 4x_1 \Longrightarrow x_2 = x_1.$$

The eigenvector is $\xi_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

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$$|A - \lambda I| = \begin{vmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{vmatrix} = (\lambda+2)(\lambda+2) - 1 = \lambda^2 + 4\lambda + 3 = 0 \to \lambda_1 = -1, \lambda_2 = -3.$$

For $\lambda_1 = -1$

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Longrightarrow -2x_1 + x_2 = -x_1 \Longrightarrow x_2 = x_1.$$

The eigenvector is $\xi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

For $\lambda_2 = -3$

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -3 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Longrightarrow -2x_1 + x_2 = -3x_1 \Longrightarrow x_2 = -x_1.$$

The eigenvector is $\xi_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Section 7.5 3. $A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

Solution:

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{vmatrix} = \lambda^2 - 4 + 3 = \lambda^2 - 1 = 0 \to \lambda_1 = 1, \quad \lambda_2 = -1,$$

For $\lambda_1 = 1$

$$\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Longrightarrow x_1 - x_2 = 0.$$

The eigenvector is $\xi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

For $\lambda_2 = -1$

$$\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Longrightarrow 3x_1 - x_2 = 0.$$

The eigenvector is $\xi_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

The general solution is $\boxed{X = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-t}}$