Hw14

Section 6.3 9. Solution:

$f(t) = u_2(t)(t-2)^2$.

i. $c = 2,\ f(t) = t^2$

ii. $F(s) = L(t^2) = \frac{2}{s^3}$

iii. $L(f(t)) = e^{-2s}\frac{2}{s^3}$.

14. Solution:

$F(s) = \frac{e^{-2s}}{s^2+s-2}$.

i. $c = 2,\ F(s) = \frac{1}{s^2+s-2}$

ii. $F(s) = \frac{1}{s^2+s-2} = \frac{1}{3}\frac{1}{s-1} - \frac{1}{3}\frac{1}{s+2}$

$f(t) = L^{-1}(F(s)) = \frac{1}{3}L^{-1}(\frac{1}{s-1}) - \frac{1}{3}L^{-1}(\frac{1}{s+2}) = \frac{1}{3}(e^t - e^{-2t})$

iii. $L^{-1}(F(s)) = \frac{1}{3}u_2(t)(e^{t-2} - e^{-2(t-2)})$.

Section 6.4 1(a). Solution:

$f(t) = u_0(t) - u_{3\pi}(t)$

Let $Y(s) = L(y(t))$, we have

$L(f(t)) = L(u_0(t)) - L(u_{3\pi}(t)) = \frac{1}{s} - e^{-3\pi s}\frac{1}{s}$.

$L(y'' + y)) = s^2Y(s) - sy(0) - y'(0) + Y(s) = (s^2+1)Y(s) - 1$

$$(s^2+1)Y(s) = 1 + \frac{1}{s} - e^{-3\pi s}\frac{1}{s} \Longrightarrow$$

$$Y(s) = \frac{1}{s(s^2+1)} - \frac{e^{-3\pi s}}{s(s^2+1)} + \frac{1}{(s^2+1)}$$

$$\begin{aligned}\frac{1}{s(s^2+1)} &= \frac{As+B}{s^2+1} + \frac{C}{s}\\ &= \frac{As^2+Bs+Cs^2+C}{(s^2+1)s}\\ &= \frac{(A+C)s^2+Bs+C}{(s^2+1)s}\end{aligned}$$

We have $\begin{cases} A+C & = & 0 \\ B & = & 0 \\ C=1 \end{cases} \Longrightarrow \begin{cases} A & = & -1 \\ B & = & 0 \\ C & = & 1 \end{cases}$

Therefore

$\frac{1}{s(s^2+1)} = -\frac{s}{s^2+1} + \frac{1}{s}$.

$$L^{-1}(\frac{1}{s(s^2+1)}) = L^{-1}(-\frac{s}{s^2+1} + \frac{1}{s}) = -\cos t + 1,$$

$$L^{-1}(\frac{e^{-3\pi s}}{s(s^2+1)}) = u_{3\pi}(-\cos(t-3\pi)+1),$$

and

$$L^{-1}(\frac{1}{(s^2+1)}) = \sin t$$

.

$y(t) = -\cos t + 1 + u_{3\pi}(-\cos(t-3\pi)+1) + \sin t.$