Hw3 Solution

Section 2.6

3. $(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)y' = 0$
Solution: $M(x,y) = 3x^2 - 2xy + 2,\ N(x,y) = 6y^2 - x^2 + 3$
$\frac{\partial M}{\partial y} = -2x, \quad \frac{\partial N}{\partial x} = -2x = \frac{\partial M}{\partial y} \Longrightarrow$ it is exact.

$$\begin{aligned}
\psi(x,y) &= \int M(x,y)dx = \int 3x^2 - 2xy + 2dx = x^3 - x^2y + 2x + C_1(y),\\
\psi(x,y) &= \int N(x,y)dy = \int 6y^2 - x^2 + 3dy = 2y^3 - x^2y + 3y + C_2(x),\\
&\Longrightarrow\\
C_1(y) &= 2y^3 + 3y,\\
C_2(x) &= x^3 + 2x,\\
&\Longrightarrow\\
\psi(x,y) &= x^3 - x^2y + 2x + 2y^3 + 3y.
\end{aligned}$$

Therefore the solution is

$$\psi(x,y) = C \Longrightarrow x^3 - x^2y + 2x + 2y^3 + 3y = C.$$

4. $\frac{dy}{dx} = -\frac{ax+by}{bx+cy}$
Solution: $(ax+by)dx + (bx+cy)dy = 0$
$M(x,y) = ax + by,\ N(x,y) = bx + cy$
$\frac{\partial M}{\partial y} = b, \quad \frac{\partial N}{\partial x} = b = \frac{\partial M}{\partial y} \Longrightarrow$ it is exact.

$$\begin{aligned}
\psi(x,y) &= \int M(x,y)dx = \int ax + bydx = \frac{1}{2}ax^2 + bxy + C_1(y),\\
\psi(x,y) &= \int N(x,y)dy = \int bx + cydy = bxy + \frac{1}{2}y^2 + C_2(x),\\
&\Longrightarrow\\
C_1(y) &= \frac{1}{2}y^2,\\
C_2(x) &= \frac{1}{2}x^2,\\
&\Longrightarrow\\
\psi(x,y) &= \frac{1}{2}ax^2 + bxy + \frac{1}{2}y^2.
\end{aligned}$$

Therefore the solution is

$$\psi(x,y) = C \Longrightarrow \frac{1}{2}ax^2 + bxy + \frac{1}{2}y^2 = C.$$

9. $(2x-y)+(2y-x)y'=0$
Solution: $(2x-y)dx+(2y-x)dy=0$
$M(x,y)=2x-y,\ N(x,y)=2y-x$
$\frac{\partial M}{\partial y}=-1,\quad \frac{\partial N}{\partial x}=-1=\frac{\partial M}{\partial y}\Longrightarrow$ it is exact.

$$\begin{aligned}\psi(x,y) &= \int M(x,y)dx=\int 2x-ydx=x^2-xy+C_1(y),\\ \psi(x,y) &= \int N(x,y)dy=\int 2y-xdy=-xy+y^2+C_2(x),\\ &\Longrightarrow\\ C_1(y) &= y^2,\\ C_2(x) &= x^2,\\ &\Longrightarrow\\ \psi(x,y) &= x^2-xy+y^2.\end{aligned}$$

Therefore the solution is

$$\psi(x,y)=C\Longrightarrow x^2-xy+y^2=C.$$

Since $y(1)=3$, we have $1-3+9=C$ and $x^2-xy+y^2-7=0$.
Therefore

$$y=\frac{x\pm\sqrt{x^2-4(x^2-7)}}{2}=\frac{x\pm\sqrt{-3x^2+28}}{2}.$$

Since $y(1)=3$, we only take "+" and $y=\frac{x+\sqrt{x^2-4(x^2-7)}}{2}=\frac{x\pm\sqrt{-3x^2+28}}{2}$. This requires that $-3x^2+28\geq 0\Longrightarrow |x|\leq\sqrt{\frac{28}{3}}$.

11. $(xy^2+bx^2y)+((x+y)x^2)y'=0$
Solution: $M(x,y)=xy^2+bx^2y,\ N(x,y)=(x+y)x^2$
$\frac{\partial M}{\partial y}=2xy+bx^2,\quad \frac{\partial N}{\partial x}=x^2+2(x+y)x=3x^2+2xy=\frac{\partial M}{\partial y}\Longrightarrow b=3$ it is exact.

$$\begin{aligned}\psi(x,y) &= \int M(x,y)dx=\int xy^2+3x^2ydx=\frac{1}{2}x^2y^2+x^3y+C_1(y),\\ \psi(x,y) &= \int N(x,y)dy=\int(x+y)x^2dy=x^3y+\frac{1}{2}x^2y^2+C_2(x),\\ &\Longrightarrow\\ C_1(y) &= 0,\\ C_2(x) &= 0,\\ &\Longrightarrow\\ \psi(x,y) &= \frac{1}{2}x^2y^2+x^3y.\end{aligned}$$

Therefore the solution is

$$\psi(x, y) = C \Longrightarrow \frac{1}{2}x^2y^2 + x^3y = C.$$

14. Proof: $M(x, y) = M(x)$, $N(x, y) = N(y)$
$\frac{\partial M}{\partial y} = 0, \quad \frac{\partial N}{\partial x} = 0 = \frac{\partial M}{\partial y} \Longrightarrow$ it is exact.