

Hw6 Solution

Section 3.1 7. $y'' + y' - 2y = 0$ $y(0) = 1, y'(0) = 1$

Solution: $a = 1, b = 1, c = -2.$

$r^2 + r - 2 = 0 \implies r = 1$ or $r = -2.$ Therefore

$$y(t) = C_1 e^t + C_2 e^{-2t} \quad y'(t) = C_1 e^t - 2C_2 e^{-2t}.$$

$$y(0) = 1 \implies C_1 + C_2 = 1$$

$$y'(0) = 1 \implies C_1 - 2C_2 = 1$$

\implies

$$C_1 = 1 \quad C_2 = 0$$

$y(t) = e^t \quad y \rightarrow +\infty$ as $t \rightarrow +\infty.$

9. $y'' + 3y' = 0,$ $y(0) = -2, y'(0) = 3.$

Solution: $a = 1, b = 3, c = 0.$

$r^2 + 3r = 0 \implies r = 0$ or $r = -3.$ Therefore

$$y(t) = C_1 + C_2 e^{-3t} \quad y'(t) = -3C_2 e^{-3t}.$$

$$y(0) = -2 \implies C_1 + C_2 = -2$$

$$y'(0) = 3 \implies -3C_2 = 3$$

\implies

$$C_1 = -1 \quad C_2 = -1$$

$y(t) = -1 - e^{-3t} \quad y \rightarrow -1$ as $t \rightarrow +\infty.$

12. $4y'' - y = 0,$ $y(-2) = 1, y'(-2) = -1.$

Solution: $a = 4, b = 0, c = -1.$

$4r^2 - 1 = 0 \implies r = 1/2$ or $r = -1/2.$ Therefore

$$y(t) = C_1 e^{t/2} + C_2 e^{-t/2} \quad y'(t) = C_1/2 e^{t/2} - C_2/2 e^{-t/2}.$$

$$\begin{aligned}
y(-2) = 1 &\implies C_1e^{-1} + C_2e = 1 \\
y'(-2) = -1 &\implies C_1/2e^{-1} - C_2/2e = -1 \\
&\implies \\
&C_1 = -e/2 \quad C_2 = 3e^{-1}/2
\end{aligned}$$

$$y(t) = -e/2e^{t/2} + 3/2e^{-1}e^{-t/2} \quad y \rightarrow -\infty \text{ as } t \rightarrow +\infty.$$

14. $y'' - y = 0, \quad y(0) = 5/4, y'(0) = -3/4.$

Solution: $a = 1, b = 0, c = -1.$

$r^2 - 1 = 0 \implies r = 1 \text{ or } r = -1.$ Therefore

$$y(t) = C_1e^t + C_2e^{-t} \quad y'(t) = C_1e^t - C_2e^{-t}.$$

$$\begin{aligned}
y(0) = 5/4 &\implies C_1 + C_2 = 5/4 \\
y'(0) = -3/4 &\implies C_1 - C_2 = -3/4 \\
&\implies \\
&C_1 = 1/4 \quad C_2 = 1
\end{aligned}$$

$$y(t) = e^t/4 + e^{-t}$$

$$\begin{aligned}
y'(t) &= e^t/4 - e^{-t} = 0 \implies e^{2t} = 4 \implies t = \ln 2 \\
y''(\ln 2) &= 1/4 \times 2 + 1/2 > 0.
\end{aligned}$$

Therefore, its minimum value is $y(\ln 2) = 1/4 \times 2 + 1/2 = 1.$

16. $y'' - y' - 2y = 0, \quad y(0) = \alpha, y'(0) = 2.$

Solution: $a = 1, b = -1, c = -2.$

$r^2 - r - 2 = 0 \implies r = 2 \text{ or } r = -1.$ Therefore

$$y(t) = C_1e^{2t} + C_2e^{-t} \quad y'(t) = 2C_1e^{2t} - C_2e^{-t}.$$

$$\begin{aligned}
y(0) = \alpha &\implies C_1 + C_2 = \alpha \\
y'(0) = 2 &\implies 2C_1 - C_2 = 2 \\
&\implies \\
&C_1 = \frac{\alpha + 2}{3} \quad C_2 = \frac{2\alpha - 2}{3}.
\end{aligned}$$

$$y(t) = \frac{\alpha+2}{3}e^{2t} + \frac{2\alpha-2}{3}e^{-t}$$

Since we want $y \rightarrow 0$ as $t \rightarrow +\infty$, we have

$$\frac{\alpha+2}{3} = 0 \implies \alpha = -2.$$