Hw7 Solution

Section 3.2 3. Solution:

$$y_1 = e^{-2t}, \qquad y_2 = te^{-2t}$$
$$y_1' = -2e^{-2t} \qquad y_2' = e^{-2t} - 2te^{-2t}.$$

Therefore

$$\begin{aligned} W &= y_1 y_2' - y_1' y_2 = e^{-2t}(e^{-2t} - 2te^{-2t}) + 2e^{-2t}te^{-2t} \\ &= e^{-4t}. \end{aligned}$$

4. Solution:

$$y_1 = e^t \sin t, \qquad y_2 = e^t \cos t$$
$$y_1' = e^t \sin t + e^t \cos t \qquad y_2' = e^t \cos t - e^t \sin t.$$

Therefore

$$\begin{aligned} W &= y_1 y_2' - y_1' y_2 = e^t \sin t \left(e^t \cos t - e^t \sin t\right) - \left(e^t \sin t + e^t \cos t\right) e^t \cos t \\ &= e^{2t} \left(\sin t \cos t - \sin^2 t - \sin t \cos t - \cos^2 t\right) = -e^{2t}. \end{aligned}$$

19. Solution:

$$y_1 = \cos(2t), \qquad y_2 = \sin(2t)$$
$$y_1' = -2\sin(2t) \qquad y_2' = 2\cos(2t)$$
$$y_1'' = -4\cos(2t) \qquad y_2' = -4\sin(2t)$$

$$\begin{aligned} y_1'' + 4y_1 &= -4\cos(2t) + 4\cos(4t) = 0 \\ y_2'' + 4y_2 &= -4\sin(2t) + 4\sin(4t) = 0 \end{aligned}$$

Therefore y_1 and y_2 are solutions of the equation $y'' + 4y = 0$.

$$\begin{aligned} W &= y_1 y_2' - y_1' y_2 = \cos(2t) 2\cos(2t) + 2\sin(2t)\sin(2t) \\ &= 2(\cos^2(2t) + \sin^2(2t)) = 2 \neq 0. \end{aligned}$$

They constitute a fundemental set of solutions.

20. Solution:

$$\begin{array}{ll} y_1 = e^t, & y_2 = te^t \\ y_1' = e^t & y_2' = e^t + te^t \\ y_1'' = e^t & y_2'' = 2e^t + te^t \end{array}$$

$$\begin{aligned} y_1'' - 2y_1' + y_1 &= e^t - 2e^t + e^t = 0 \\ y_2'' - 2y_2' + y_2 &= 2e^t + te^t - 2(e^t + te^t) + te^t = 0 \end{aligned}$$

Therefore y_1 and y_2 are solutions of the equation $y'' - 2y' + y = 0$.

$$W = y_1 y_2' - y_1' y_2 = e^t(2e^t + te^t) - e^t te^t = 2e^{2t} \neq 0.$$

They constitute a fundemental set of solutions.

23. Solution:

$$y'' - \frac{t+2}{t}y' + \frac{t+2}{t^2}y = 0.$$

$$p(t) = -\frac{t+2}{t}.$$

$$W = Ce^{-\int p(t)dt} = Ce^{\int \frac{t+2}{t}dt} = Ce^{\int 1+\frac{2}{t}dt} = Ce^{t+2\ln|t|} = Ct^2e^t.$$