Hw8 Solution

Section 3.3

5. $y'' - 2y' + 2y = 0$

$\boxed{\text{Solution:}}$

$r^2 - 2r + 2 = 0 \Longrightarrow r = \frac{2\pm\sqrt{-4}}{2} = 1 \pm i.$

We have $\lambda = 1, \quad \mu = 1$ and

$$y_1(t) = e^{\lambda t}\cos\mu t = e^t \cos t \quad y_2(t) = e^{\lambda t}\sin\mu t = e^t \sin t.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^t \cos t + C_2 e^t \sin t.}$

9. $y'' + 2y' + 1.25y = 0$

$\boxed{\text{Solution:}}$

$r^2 + 2r + 1.25 = 0 \Longrightarrow r = \frac{-2\pm\sqrt{-1}}{2} = -1 \pm i/2.$

We have $\lambda = -1, \quad \mu = 1/2$ and

$$y_1(t) = e^{\lambda t}\cos\mu t = e^{-t} \cos t/2 \quad y_2(t) = e^{\lambda t}\sin\mu t = e^{-t} \sin t/2.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^{-t} \cos t/2 + C_2 e^{-t} \sin t/2.}$

12. $y'' + 4y = 0, \quad y(0) = 0, y'(0) = 1$

$\boxed{\text{Solution:}}$

$r^2 + 4 = 0 \Longrightarrow r = \frac{\pm\sqrt{-16}}{2} = \pm 2i.$

We have $\lambda = 0, \quad \mu = 2$ and

$$y_1(t) = e^{\lambda t}\cos\mu t = \cos 2t \quad y_2(t) = e^{\lambda t}\sin\mu t = \sin 2t.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 \cos 2t + C_2 \sin 2t.}$

$y' = -2C_1 \sin 2t + 2C_2 \cos 2t.$

$$\begin{aligned} y(0) = 0 &\Longrightarrow C_1 = 0 \\ y'(0) = 1 &\Longrightarrow 2C_2 = 1 \\ &\Longrightarrow \\ & \quad C_1 = 0 \quad C_2 = 1/2 \end{aligned}$$

$\boxed{y(t) = \frac{1}{2}\sin 2t. \text{ Steady oscillation for increasing } t.}$

13. $y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, y'(\pi/2) = 1$

$\boxed{\text{Solution:}}$

$r^2 - 2r + 5 = 0 \Longrightarrow r = \frac{2 \pm \sqrt{-16}}{2} = 1 \pm 2i.$

We have $\lambda = 1, \quad \mu = 2$ and

$$y_1(t) = e^{\lambda t} \cos \mu t = e^t \cos 2t \quad y_2(t) = e^{\lambda t} \sin \mu t = e^t \sin 2t.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^t \cos 2t + C_2 e^t \sin 2t.}$

$y' = -2C_1 e^t(\cos 2t - \sin 2t) + 2C_2 e^t(\sin 2t + \cos 2t).$

$$\begin{aligned} y(\pi/2) = 0 &\implies -C_1 e^{\pi/2} = 0 \\ y'(\pi/2) = 2 &\implies -C_1 e^{\pi/2} - 2C_2 e^{\pi/2} = 2 \\ &\implies \\ & \quad C_1 = 0 \quad C_2 = -e^{-\pi/2} \end{aligned}$$

$\boxed{y(t) = -e^{-\pi/2} e^2 \sin 2t. \text{ growing oscillation for increasing } t.}$