Problems

In each of Problems 1 through 4, determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. $t^2\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = \sin t$

2. $(1+y^2)\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + y = e^t$

3. $\dfrac{d^4y}{dt^4} + \dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 1$

4. $\dfrac{d^2y}{dt^2} + \sin(t+y) = \sin t$

In each of Problems 5 through 10, verify that each given function is a solution of the differential equation.

5. $y'' - y = 0; \quad y_1(t) = e^t, \ y_2(t) = \cosh t$
6. $y'' + 2y' - 3y = 0; \quad y_1(t) = e^{-3t}, \ y_2(t) = e^t$
7. $ty' - y = t^2; \quad y = 3t + t^2$
8. $y'''' + 4y''' + 3y = t; \quad y_1(t) = t/3, \ y_2(t) = e^{-t} + t/3$
9. $t^2y'' + 5ty' + 4y = 0, \ t > 0; \quad y_1(t) = t^{-2}, \ y_2(t) = t^{-2}\ln t$
10. $y' - 2ty = 1; \quad y = e^{t^2}\displaystyle\int_0^t e^{-s^2}\,ds + e^{t^2}$

In each of Problems 11 through 13, determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$.

11. $y' + 2y = 0$
12. $y'' + y' - 6y = 0$
13. $y''' - 3y'' + 2y' = 0$

In each of Problems 14 and 15, determine the values of r for which the given differential equation has solutions of the form $y = t^r$ for $t > 0$.

14. $t^2y'' + 4ty' + 2y = 0$
15. $t^2y'' - 4ty' + 4y = 0$

In each of Problems 16 through
partial differential equation; also
or nonlinear. Partial derivatives a

16. $u_{xx} + u_{yy} + u_{zz} = 0$
17. $u_{xxxx} + 2u_{xxyy} + u_{yyyy} =$
18. $u_t + uu_x = 1 + u_{xx}$

In each of Problems 19 through
a solution of the given partial

19. $u_{xx} + u_{yy} = 0; \quad u_1(x,$
$u_2(x,y) = \ln(x^2+y^2)$
20. $\alpha^2 u_{xx} = u_t; \quad u_1(x,t)$
$u_2(x,t) = e^{-\alpha^2\lambda^2 t}\sin\lambda x,$
21. $a^2 u_{xx} = u_{tt}; \quad u_1(x,t$
$u_2(x,t) = \sin(x - at), \ \lambda$
22. Follow the steps indica
of a pendulum, equation (12
and weightless, that the m
friction or drag anywhere i
 a. Assume that the r
 indicated by the angle
 forces acting on the m
 b. Apply Newton's
 the circular arc on wh
 in the rod does not e
 find the component
 direction. Observe a
 the angular accelera
 the rod.
 c. Simplify the res
 text.