Problems

In each of Problems 1 through 8:

G **a.** Draw a direction field for the given differential equation.
b. Based on an inspection of the direction field, describe how solutions behave for large t.
c. Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$.

1. $y' + 3y = t + e^{-2t}$
2. $y' - 2y = t^2 e^{2t}$
3. $y' + y = te^{-t} + 1$
4. $y' + \frac{1}{t}y = 3\cos(2t), \quad t > 0$
5. $y' - 2y = 3e^t$
6. $ty' - y = t^2 e^{-t}, \quad t > 0$
7. $y' + y = 5\sin(2t)$
8. $2y' + y = 3t^2$

In each of Problems 9 through 12, find the solution of the given initial value problem.

9. $y' - y = 2te^{2t}, \quad y(0) = 1$
10. $y' + 2y = te^{-2t}, \quad y(1) = 0$
11. $y' + \frac{2}{t}y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, \quad t > 0$
12. $ty' + (t + 1)y = t, \quad y(\ln 2) = 1, \quad t > 0$

In each of Problems 13 and 14:

G **a.** Draw a direction field for the given differential equation. How do solutions appear to behave as t becomes large? Does the behavior depend on the choice of the initial value a? Let a_0 be the value of a for which the transition from one type of behavior to another occurs. Estimate the value of a_0.
b. Solve the initial value problem and find the critical value a_0 exactly.
c. Describe the behavior of the solution corresponding to the initial value a_0.

13. $y' - \frac{1}{2}y = 2\cos t, \quad y(0) = a$
14. $3y' - 2y = e^{-\pi t/2}, \quad y(0) = a$