

19. Consider the initial value problem

$$y' + \frac{1}{4}y = 3 + 2 \cos(2t), \quad y(0) = 0.$$

a. Find the solution of this initial value problem and describe its behavior for large  $t$ .

**N** b. Determine the value of  $t$  for which the solution first intersects the line  $y = 12$ .

20. Find the value of  $y_0$  for which the solution of the initial value problem

$$y' - y = 1 + 3 \sin t, \quad y(0) = y_0$$

remains finite as  $t \rightarrow \infty$ .

21. Consider the initial value problem

$$y' - \frac{3}{2}y = 3t + 2e^t, \quad y(0) = y_0.$$

Find the value of  $y_0$  that separates solutions that grow positively as  $t \rightarrow \infty$  from those that grow negatively. How does the solution that corresponds to this critical value of  $y_0$  behave as  $t \rightarrow \infty$ ?

22. Show that all solutions of  $2y' + ty = 2$  [equation (41) of the text] approach a limit as  $t \rightarrow \infty$ , and find the limiting value.

*Hint:* Consider the general solution, equation (47). Show that the first