Problems

Determine whether each of the equations in Problems 1 through 8 is exact. If it is exact, find the solution.

1. $(2x + 3) + (2y - 2)\,y' = 0$
2. $(2x + 4y) + (2x - 2y)\,y' = 0$
3. $(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)\,y' = 0$
4. $\dfrac{dy}{dx} = -\dfrac{ax + by}{bx + cy}$
5. $\dfrac{dy}{dx} = -\dfrac{ax - by}{bx - cy}$
6. $(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x) + (xe^{xy}\cos(2x) - 3)\,y' = 0$
7. $(y/x + 6x) + (\ln x - 2)\,y' = 0, \quad x > 0$
8. $\dfrac{x}{(x^2 + y^2)^{3/2}} + \dfrac{y}{(x^2 + y^2)^{3/2}}\dfrac{dy}{dx} = 0$

In each of Problems 9 and 10, solve the given initial value problem and determine at least approximately where the solution is valid.

9. $(2x - y) + (2y - x)\,y' = 0, \quad y(1) = 3$
10. $(9x^2 + y - 1) - (4y - x)\,y' = 0, \quad y(1) = 0$

In each of Problems 11 and 12, find the value of b for which the given equation is exact, and then solve it using that value of b.

11. $(xy^2 + bx^2y) + (x + y)x^2y' = 0$
12. $(ye^{2xy} + x) + bxe^{2xy}\,y' = 0$
13. Assume that equation (6) meets the requirements of Theorem 2.6.1 in a rectangle R and is therefore exact. Show that a possible function $\psi(x, y)$ is

$$\psi(x, y) = \int_{x_0}^{x} M(s, y_0)\,ds + \int_{y_0}^{y} N(x, t)\,dt,$$

where (x_0, y_0) is a point in R.

14. Show that any separable equation

$$M(x) + N(y)\,y' = 0$$

is also exact.

In each of Problems 15 and 16, show that the given equation is not exact but becomes exact when multiplied by the given integrating factor. Then solve the equation.

15. $x^2y^3 + x(1 + y^2)\,y' = 0, \quad \mu(x, y) = 1/\left(xy^3\right)$
16. $(x + 2)\sin y + (x\cos y)\,y' = 0, \quad \mu(x, y) = xe^x$
17. Show that if $(N_x - M_y)/M = Q$, where Q is a function of y only, then the differential equation

$$M + Ny' = 0$$

has an integrating factor of the form

$$\mu(y) = \exp\int Q(y)dy.$$

In each of Problems 18 through 21, find an integrating factor and solve the given equation.

18. $(3x^2y + 2xy + y^3) + (x^2 + y^2)\,y' = 0$
19. $y' = e^{2x} + y - 1$
20. $1 + (x/y - \sin y)\,y' = 0$
21. $y + (2xy - e^{-2y})\,y' = 0$
22. Solve the differential equation

$$(3xy + y^2) + (x^2 + xy)\,y' = 0$$

using the integrating factor $\mu(x, y) = (xy(2x + y))^{-1}$. Verify that the solution is the same as that obtained in Example 4 with a different integrating factor.