

Hw 10

Sel 4.3

2. $W = \{(x, y, 2x-3y) : x \text{ and } y \text{ are real numbers}\}$

Solution: for $U = (x_1, y_1, 2x_1-3y_1)$, $V = (x_2, y_2, 2x_2-3y_2) \in W$

$$U+V = (x_1+x_2, y_1+y_2, 2(x_1+x_2)-3(y_1+y_2)) \in W$$

$$cU = (cx_1, cy_1, 2cx_1-3cy_1) \in W$$

Therefore W is a subspace.

8. $W = \{(x, 1) : x \text{ is a real number}\}$

Solution: W is not a subspace because $(1, 1), (2, 1) \in W$

$$(1, 1) + (2, 1) = (3, 2) \notin W$$

12. W is not a subspace because $x, -x \in W$, but $x + (-x) = 0 \notin W$

30. $W = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \right\}$ W is a subspace since

$$U = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}, V = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} \text{ in } W. \quad U+V = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix} \in W$$

$$cU = \begin{bmatrix} ca_1 & 0 \\ 0 & cb_1 \end{bmatrix} \in W$$

34. W is not a subspace since

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \in W. \quad \text{but} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ is not invertible} \notin W$$

See 4.4

$$8. \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = c_1 A + c_2 B = \begin{bmatrix} 2c_1 & -3c_1 \\ 4c_1 & c_1 \end{bmatrix} + \begin{bmatrix} 0 & 5c_2 \\ c_2 & -2c_2 \end{bmatrix} = \begin{bmatrix} 2c_1 & -3c_1 + 5c_2 \\ 4c_1 + c_2 & c_1 - 2c_2 \end{bmatrix} \Rightarrow \begin{cases} 2c_1 = 0 \\ -3c_1 + 5c_2 = 0 \\ 4c_1 + c_2 = 0 \\ c_1 - 2c_2 = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = 0 \end{cases}$$

therefore $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \cdot A + 0 \cdot B$ is a linear combination of A, B

10. For any vector $(x_1, x_2) \in \mathbb{R}^2$

$$(x_1, x_2) = c_1(1, -1) + c_2(2, 1) = (c_1 + 2c_2, -c_1 + c_2)$$

$$\Rightarrow (x) \begin{cases} c_1 + 2c_2 = x_1 \\ -c_1 + c_2 = x_2 \end{cases} \text{ the coefficient matrix } A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \det A = 1+2=3 \neq 0$$

therefore (x) always has a unique solution and S spans \mathbb{R}^2

$$30. c_1(3, -6) + c_2(-1, 2) = 0 \Rightarrow (3c_1 - c_2, -6c_1 + 2c_2) = 0 \Rightarrow \begin{cases} 3c_1 - c_2 = 0 \\ -6c_1 + 2c_2 = 0 \end{cases} \text{ take } \begin{cases} c_1 = 1 \\ c_2 = 3 \end{cases}$$

therefore $(3, -6) + 3(-1, 2) = 0$ and S is linearly dependent

$$42. c_1(x^2-1) + c_2(2x+5) = 0 \Rightarrow c_1x^2 + 2c_2x + (-c_1 + 5c_2) = 0 \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = 0 \\ -c_1 + 5c_2 = 0 \end{cases} \Rightarrow c_1 = c_2 = 0$$

therefore S is linearly independent

$$46. c_1 A + c_2 B + c_3 C = 0 \Rightarrow \begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & c_2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c_3 & 0 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_3 & 0 \end{bmatrix} = 0 \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{cases}$$

therefore A, B, C are linearly independent.