

## Hw 11

Sec 4.5

8.  $S = \{(1, 2), (1, -2), (2, 4)\}$

Solution:  $(-1, 2) + (1, -2) + 0(2, 4) = 0$  therefore  $S$  is linearly dependent  
 $S$  is not a basis

14.  $S = \{(-1, 2)\}$

Solution:  $(1, 0)$  cannot be written as the linear combination of  $(-1, 2)$   
therefore  $S$  does not span  $\mathbb{R}^2$  and  $S$  is not a basis

18.  $S = \{(1, 1, 2), (0, 2, 1)\}$

Solution:  $(1, 0, 0)$  is not in  $\text{span}(S)$  since  $(1, 0, 0) = c_1(1, 1, 2) + c_2(0, 2, 1)$   
 $= (c_1, c_1+2c_2, 2c_1+c_2)$

$$\begin{aligned} c_1 &= 1 \\ c_1 + 2c_2 &= 0 \Rightarrow c_2 = -\frac{1}{2} \\ 2c_1 + c_2 &= 2 - \frac{1}{2} \neq 0 \end{aligned}$$

therefore  $S$  doesn't span  $\mathbb{R}^3$  and  $S$  is not a basis

22.  $S = \{2, x, x+3, 3x^2\}$

Solution:  $3 \cdot 2 + 2x + (-2)(x+3) + 0 \cdot 3x^2 = 0$  therefore  $S$  is linearly dependent and  $S$  is not a basis

26.  $S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$

Solution:  $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$  is not in  $\text{span}(S)$ , therefore  $\text{span}(S) \neq M_{2,2}$  and  $S$  is not a basis

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28.  $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

Solution  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} c_1 + c_3 = 1 \\ c_2 + c_3 = 2 \\ c_2 = 1 \\ c_1 = 1 \end{array} \Rightarrow \begin{array}{l} c_3 = 0 \\ c_3 = 1 \end{array}$  contradiction

therefore  $\text{Span}(S) \neq M_{2 \times 2}$  and  $S$  is not a basis

34.  $S = \{(1, 2), (1, -1)\}$  for  $\mathbb{R}^2$

Solution: ① Given any vector  $(x, y)$  in  $\mathbb{R}^2$ .  $(x, y) = c_1(1, 2) + c_2(1, -1) = (c_1 + c_2, 2c_1 - c_2)$

②  $\begin{cases} c_1 + c_2 = x \\ 2c_1 - c_2 = y \end{cases}$  the coefficient matrix  $A = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$   $\det A = -1 - 2 = -3 \neq 0$   
 $\Leftrightarrow$  has a unique solution  $\text{Span}(S) = \mathbb{R}^2$

③  $c_1(1, 2) + c_2(1, -1) = 0 \Rightarrow \begin{cases} c_1 + c_2 = 0 \\ 2c_1 - c_2 = 0 \end{cases} \det A \neq 0 \quad c_1 = c_2 = 0$   
 $S$  is linearly independent

therefore  $S$  is a basis for  $\mathbb{R}^2$

44.  $S = \{t^3 - 1, 2t^2, t + 3, 5 + 2t + 2t^2 + t^3\}$

Solution  $c_1(t^3 - 1) + c_2(2t^2) + c_3(t + 3) + c_4(5 + 2t + 2t^2 + t^3) = 0$   
 $(-c_1 + 3c_3 + 5c_4) + (c_2 + 2c_4)t + (2c_2 + 2c_4)t^2 + (c_1 + c_4)t^3 = 0$

$\begin{cases} -c_1 + 3c_3 + 5c_4 = 0 \\ c_2 + 2c_4 = 0 \\ 2c_2 + 2c_4 = 0 \\ c_1 + c_4 = 0 \end{cases}$  therefore (\*) has nontrivial solution  
 and  $S$  is linearly dependent

$S$  is not basis for  $P_3$

(3)

52.  $R$ 

$$\text{a basis } S = \{1\} \quad \dim(R) = 1$$

54.  $P_4$ 

$$\text{a basis } S = \{1, x, x^2, x^3, x^4\} \quad \dim P_4 = 5$$