Hw13

Sec 6.1

2. $v=(0,6)$ $T(0,6)=(2\cdot 6-0,\ 0,\ 6)=(12,\ 0,\ 6)$ the image of $(0,6)$ is $(12,\ 0,\ 6)$

$T(v_1,v_2)=w=(3,1,2)$ $(2v_2-v_1,\ v_1,\ v_2)=(3,\ 1,\ 2) \Rightarrow v_1=1,\ v_2=2$ the preimage of $(3,1,2)$ is $(1,2)$

6. $v=(2,1,4)$ $T(v)=(2\cdot 2+1;\ 2-1)=(5,1)$ the image of $(2,1,4)$ is $(5,1)$

$T(v_1,v_2,v_3)=(-1,2)$ $(2v_1+v_2,\ v_1-v_2)=(-1,2)$ $\begin{matrix} 2v_1+v_2=-1 \\ v_1-v_2=2 \end{matrix} \Rightarrow \begin{matrix} v_1=\frac{1}{3} \\ v_2=\frac{-5}{3} \end{matrix}$

the preimage of $w=(-1,2)$ is $\{(\frac{1}{3},\ -\frac{5}{3},\ t)$ t is any real number$\}$

10. Let $u=(x_1,y_1)$, $v=(x_2,y_2)$ $T(u+v)=T(x_1+x_2,\ y_1+y_2)=((x_1+x_2)^2,\ y_1+y_2)$

$T(u)+T(v)=(x_1^2,\ y_1)+(x_2^2,\ y_2)=(x_1^2+x_2^2,\ y_1+y_2) \neq T(u+v)$

T is not a linear transformation

18. $A=\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ $B=\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ $T(A+B)=T\begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix}=(b_1+b_2)^2$

$T(A)+T(B)=b_1+b_2 \neq T(A+B)$

T is not a linear transformation

34. $A_{3\times 2}$ $n=2$ $m=3$ $T: R^2 \to R^3$

36. $A_{2\times 5}$ $n=5$ $m=2$ $T: R^5 \to R^2$

Sec 6.2

2. $T(x,y,z) = (x, 0, z) = (0,0,0) \Rightarrow x=0,\ z=0$

$\ker(T) = \{(0, y, 0) \mid y \text{ is any real number}\}$

16. $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 0 \end{array}\right]$

$\begin{aligned} x_1 + x_2 &= 0 \\ 3x_2 &= 0 \\ x_2 &= 0 \end{aligned} \Rightarrow x_1 = x_2 = 0$

$\ker(T) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

the range of T $\quad x_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $\quad$ the basis is $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

20. $A = \begin{bmatrix} 3 & 2 \\ -9 & -6 \end{bmatrix}$ $\quad \left[\begin{array}{cc|c} 3 & 2 & 0 \\ -9 & -6 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 3 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\quad 3x_1 + 2x_2 = 0$ $\quad x_2 = 3t$, $x_1 = -2t$

$\ker(T) = \left\{ t \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right\}$ $\quad$ nullity $(T) = 1$

$\begin{bmatrix} 3 & 2 \\ -9 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ -3 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ -3 & 0 \end{bmatrix}$

$\text{range}(T) = \left\{ t \begin{bmatrix} 1 \\ -3 \end{bmatrix} \right\}$ $\quad$ rank $(T) = 1$

22. $A = \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & -3 \end{bmatrix}$ $\quad \left[\begin{array}{cc|c} 4 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & -3 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 4 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{7}{2} & 0 \end{array}\right]$ $\quad \begin{aligned} 4x_1 + x_2 &= 0 \\ -\tfrac{7}{2} x_2 &= 0 \end{aligned} \Rightarrow x_1 = x_2 = 0$

$\ker(T) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ $\quad$ nullity $= 0$

$\text{range}(T) = \left\{ t \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \right\}$ $\quad$ rank $(T) = 2$

t, s any real number