HW2

Sec 1.2

2. 4×1 6. 1×5

12. $x_1 = 2$
$x_2 = 3$

14 $x_1 + 2x_2 + x_3 = 0$
$x_3 = -1$

26. $\begin{pmatrix} 2 & 6 & 16 \\ -2 & -6 & -16 \end{pmatrix} \to \begin{pmatrix} 2 & 6 & 16 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 8 \\ 0 & 0 & 0 \end{pmatrix}$ $x_1 + 3x_2 = 8$
$x_1 = 8 - 3x_2$

$x_2 = 5$
$x_1 = 8 - 3(5)$

the solution is $\begin{cases} x_1 = 8-3(5) \\ x_2 = 5 \end{cases}$

28. $\begin{pmatrix} 2 & -1 & -0.1 \\ 3 & 2 & 1.6 \end{pmatrix} \to \begin{pmatrix} 2 & -1 & -0.1 \\ 0 & 3.5 & 1.75 \end{pmatrix}$ $3.5x_2 = 1.75$ $x_2 = 0.5$

$2x_1 - x_2 = -0.1 \Rightarrow x_1 = 0.2$

the solution is $\begin{cases} x_1 = 0.2 \\ x_2 = 0.5 \end{cases}$

30 $\begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 6 \\ 3 & -2 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 6 \\ 0 & -8 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 6 \\ 0 & 0 & -40 \end{pmatrix}$ $0 = -40$ no solution

Sec 1.3.

2. $(0,0)$, $(2,-2)$, $(4,0)$

$n=3$.

$p(x) = a_0 + a_1 x + a_2 x^2$

$p(0)=0 \Rightarrow a_0 = 0$

$p(2)=-2 \Rightarrow a_0 + 2a_1 + 4a_2 = -2$

$p(4)=0 \quad a_0 + 4a_1 + 16a_2 = 0$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & -2 \\ 1 & 4 & 16 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & -2 \\ 0 & 4 & 16 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 4 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$2a_2 = 1 \quad a_2 = \frac{1}{2}$

$a_1 + 2a_2 = -1 \quad a_1 = -1 - 2a_2 = -2$

$a_0 = 0$

$p(x) = -2x + \frac{1}{2}x^2$

10. $(2005, 150)$ $(2006, 180)$ $(2007, 240)$ $(2008, 360)$

$z = x - 2005$ $(0\ 150)$ $(1, 180)$ $(2\ 240)$ $(3\ 360)$ $n=4$

$p(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3$

$p(0)=150 \Rightarrow a_0 = 150$

$p(1)=180 \Rightarrow a_0 + a_1 + a_2 + a_3 = 180$

$p(2)=240 \Rightarrow a_0 + 2a_1 + 4a_2 + 8a_3 = 240$

$p(3)=360 \Rightarrow a_0 + 3a_1 + 9a_2 + 27a_3 = 360$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 150 \\ 1 & 1 & 1 & 1 & 180 \\ 1 & 2 & 4 & 8 & 240 \\ 1 & 3 & 9 & 27 & 360 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 & 150 \\ 0 & 1 & 1 & 1 & 30 \\ 0 & 2 & 4 & 8 & 90 \\ 0 & 3 & 9 & 27 & 210 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 & 150 \\ 0 & 1 & 1 & 1 & 30 \\ 0 & 0 & 2 & 6 & 30 \\ 0 & 0 & 6 & 24 & 120 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 & 150 \\ 0 & 1 & 1 & 1 & 30 \\ 0 & 0 & 1 & 3 & 15 \\ 0 & 0 & 1 & 4 & 20 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 & 150 \\ 0 & 1 & 1 & 1 & 30 \\ 0 & 0 & 1 & 3 & 15 \\ 0 & 0 & 0 & 1 & 5 \end{pmatrix}$$

$a_3 = 5$

$a_2 + 3a_3 = 15 \Rightarrow a_2 = 0$

$a_1 + a_2 + a_3 = 30 \Rightarrow a_1 = 30 - a_2 - a_3 = 25$

$a_0 = 150$

$p(z) = 150 + 25z + 5z^3 \Rightarrow P(x) = 150 + 25(x-2005) + 5(x-2005)^3$

Hw 2

14 $\log_2 1 = 0$ $\log_2 2 = 1$ $\log_2 4 = 2$

$(1, 0)$ $(2, 1)$ $(4, 2)$ $n = 3$

$P(x) = a_0 + a_1 x + a_2 x^2$

$P(1) = 0 \Rightarrow a_0 + a_1 + a_2 = 0$

$P(2) = 1 \Rightarrow a_0 + 2a_1 + 4a_2 = 1$

$P(4) = 2 \Rightarrow a_0 + 4a_1 + 16a_2 = 2$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 4 & 1 \\ 1 & 4 & 16 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 15 & 2 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 6 & -1 \end{pmatrix} \to$$

$6a_2 = -1 \Rightarrow a_2 = -\frac{1}{6}$

$a_1 + 3a_2 = 1 \Rightarrow a_1 = 1 - 3a_2 = \frac{3}{2}$

$a_0 + a_1 + a_2 = 0 \Rightarrow a_0 = -a_1 - a_2 = -\frac{4}{3}$

$P(x) = -\frac{4}{3} + \frac{3}{2}x - \frac{1}{6}x^2$

$$\log_2 3 \approx P(3) = -\frac{4}{3} + \frac{3}{2}\cdot 3 - \frac{1}{6}\cdot 3^2 = -\frac{4}{3} + \frac{9}{2} - \frac{9}{6} = \frac{5}{3}$$