Hw11 Solution

Section 7.8 1(c). $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

$\boxed{\text{Solution:}}$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & -4 \\ 1 & -1-\lambda \end{vmatrix} = (\lambda - 3)(\lambda + 1) + 4 = \lambda^2 - 2\lambda + 1 = 0 \to \lambda = 1.$$

For $\lambda = 1$

$$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Longrightarrow x_1 - 2x_2 = 0.$$

The eigenvector is $\Gamma = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Longrightarrow x_1 = 2x_2 + 1.$$

$\Gamma_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. The general solution is

$$\boxed{X = C_1 e^t \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} te^t + \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^t \right)}$$

7(a). $A = \begin{pmatrix} -5/2 & 3/2 \\ -3/2 & 1/2 \end{pmatrix}$ $\quad x(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\boxed{\text{Solution:}}$

$$|A - \lambda I| = \begin{vmatrix} -5/2-\lambda & 3/2 \\ -3/2 & 1/2-\lambda \end{vmatrix} = (\lambda+5/2)(\lambda-1/2)+9/4 = \lambda^2+2\lambda+1 = 0 \to \lambda = -1.$$

For $\lambda = -1$

$$\begin{bmatrix} -3/2 & 3/2 \\ -3/2 & 3/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Longrightarrow x_1 - x_2 = 0.$$

The eigenvector is $\Gamma = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} -3/2 & 3/2 \\ -3/2 & 3/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Longrightarrow x_2 = x_1 + 2/3.$$

$\Gamma_2 = \begin{bmatrix} -2/3 \\ 0 \end{bmatrix}$. The general solution is

$$\boxed{X = C_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} te^{-t} + \begin{bmatrix} -2/3 \\ 0 \end{bmatrix} e^{-t} \right)}$$

$$\begin{aligned} x(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \quad &\Longrightarrow \quad C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} -2/3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \\ &\Longrightarrow \quad C_1 - 2/3C_2 = 3 \quad C_1 = -1 \\ &\Longrightarrow \\ &\qquad C_1 = -1 \quad C_2 = -6. \end{aligned}$$

The solution is

$$\boxed{X == -e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 6 \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} te^{-t} + \begin{bmatrix} -2/3 \\ 0 \end{bmatrix} e^{-t} \right) = e^{-t} \begin{bmatrix} 3 \\ -1 \end{bmatrix} - te^{-t} \begin{bmatrix} 6 \\ 6 \end{bmatrix}}$$

8(a). $A = \begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix} \quad x(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\boxed{\text{Solution:}}$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & 9 \\ -1 & -3-\lambda \end{vmatrix} = (\lambda - 3)(\lambda + 3) + 9 = \lambda^2 = 0 \to \lambda = 0.$$

For $\lambda = 0$

$$\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Longrightarrow x_1 + 3x_2 = 0.$$

The eigenvector is $\Gamma = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \Longrightarrow x_1 = -3x_2 - 1.$$

$\Gamma_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$. The general solution is

$$\boxed{X = C_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} + C_2 \left(\begin{bmatrix} -3 \\ 1 \end{bmatrix} t + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right)}$$

$$\begin{aligned} x(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad &\Longrightarrow \quad C_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ &\Longrightarrow \quad -3C_1 - C_2 = 2 \quad C_1 = 4 \\ &\Longrightarrow \\ &\qquad C_1 = 4 \quad C_2 = -14. \end{aligned}$$

The solution is

$$\boxed{X == 4 \begin{bmatrix} -3 \\ 1 \end{bmatrix} - 14 \left(\begin{bmatrix} -3 \\ 1 \end{bmatrix} t + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 4 \end{bmatrix} - 14t \begin{bmatrix} -3 \\ 1 \end{bmatrix}}$$