Hw12 Solution

Section 6.1 4(b). $f(t) = t^2$

Solution:

$$\begin{aligned} L\{f(t)\} &= L\left\{t^2\right\} = \int_0^\infty e^{-st} t^2 dt = \int_0^\infty (-\frac{1}{s}) t^2 de^{-st} \\ &= \int_0^\infty \frac{2}{s} t e^{-st} dt = \int_0^\infty -\frac{2}{s^2} t de^{-st} = \int_0^\infty \frac{2}{s^2} t e^{-st} dt = -\frac{2}{s^3} e^{-st}|_{t=0}^{\infty} = \frac{2}{s^3} \quad s > 0 \end{aligned}$$

6. Solution:

$$\begin{aligned} L\{f(t)\} &= L\{\cosh(bt)\} = \int_0^\infty e^{-st} \frac{e^{bt} + e^{-bt}}{2} dt = 1/2 \int_0^\infty e^{-(s-b)t} + e^{-(s+b)t} dt \\ &= 1/2 \frac{e^{(s-b)t}}{b-s}|_{t=0}^{\infty} + 1/2 \frac{e^{-(s+b)t}}{-b-s}|_{t=0}^{\infty} \\ &= 1/2 \left(\frac{1}{s-b} + \frac{1}{b+s} \right) = \frac{s}{s^2 - b^2} \quad s > |b|. \end{aligned}$$

9.

$$\begin{aligned} L\{f(t)\} &= L\{\cos(bt)\} = \int_0^\infty e^{-st} \frac{e^{ibt} + e^{-ibt}}{2} dt = 1/2 \int_0^\infty e^{-(s-ib)t} + e^{-(s+ib)t} dt \\ &= 1/2 \frac{e^{(s-ib)t}}{ib-s}|_{t=0}^{\infty} + 1/2 \frac{e^{-(s+ib)t}}{-ib-s}|_{t=0}^{\infty} \\ &= 1/2 \left(\frac{1}{s-ib} + \frac{1}{ib+s} \right) = \frac{s}{s^2 + b^2} \quad s > |b|. \end{aligned}$$