Hw2 Solution

Section 2.2 1.

$$y' = \frac{x^2}{y}$$

(1) $ydy = x^2dx$
(2) $\int ydy = \int x^2dx$
(3) $y^2/2 = x^3/3 + c \quad (y \neq 0)$

2.

$$y' + y^2 \sin x = 0$$

if $y \neq 0$
(1) -$y^{-2}dy = \sin x dx$
(2) -$\int y^{-2}dy = \int \sin x dx$
(3) $y^{-1} = -\cos x + c$ if $y \neq 0$
$y = 0$ everywhere is a solution as well.

5.

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}$$

(1) $(y + e^y)dy = (x - e^{-x})dx$
(2) $\int y + e^y dy = \int x - e^{-x}dx$
(3) $y^2/2 + e^y = x^2/2 + e^{-x} + c \quad (y + e^y \neq 0)$

6.

$$\frac{dy}{dx} = \frac{x^2}{1 + y^2}$$

(1) $(1 + y^2)dy = x^2dx$
(2) $\int 1 + y^2dy = \int x^2dx$
(3) $y + y^3/3 = x^3/3 + c$

18.

$$y' = \frac{3x^2}{3y^2 - 4}$$

(1) $(3y^2 - 4)dy = 3x^2dx$

(2) $\int 3y^2 - 4dy = \int 3x^2 dx$

(3) $y^3 - 4y = x^3 + c \quad (3y^2 - 4 \neq 0)$

(4) $y(1) = 0 \Longrightarrow 0 = 1 + c \Longrightarrow c = -1 \Longrightarrow$

$$y^3 - 4y = x^3 - 1. \tag{1}$$

If $3y^2 - 4 = 0 \Longrightarrow y = 2/\sqrt{3}$ or $y = -2/\sqrt{3}$.

When $y = 2/\sqrt{3}$, from (1) we have $x^3 = y^3 - 4y + 1 = \left(2/\sqrt{3}\right)^3 - 8/\sqrt{3} + 1 \Longrightarrow x \approx -1.28$.

When $y = -2/\sqrt{3}$, from (1) we have $x^3 = y^3 - 4y + 1 = -\left(2/\sqrt{3}\right)^3 + 8/\sqrt{3} + 1 \Longrightarrow x = 1.60$

Therefore, the interval in which the solution is valid is $(-1.28, 1.60)$.