Hw4 Solution

Section 2.3 7. Solution: Let $B(t)$ be the balance the student owes at time t. We have

$$\begin{cases} \frac{dB}{dt} &= 0.1B - k \\ B(0) &= 8000. \end{cases}$$

To pay off the loan in 3 years, we have $B(3) = 0$.

$$\frac{dB}{dt} - 0.1B = -k \Longrightarrow$$

(1) $p(t) = -0.1$, $g(t) = -k$

(2) $\mu = e^{\int p(t)dt} = e^{\int -0.1dt} = e^{-0.1t}$

(3) $y(t) = \frac{\int \mu(t)g(t)dt}{\mu(t)} = \frac{\int e^{-0.1t}(-k)dt + c}{e^{-0.1t}} = \frac{10ke^{-0.1t}+c}{e^{-0.1t}} = 10k + ce^{0.1t}$.

$B(0) = 8000 \Longrightarrow 8000 = 10k + c \Longrightarrow c = 8000 - 10k$. Therefore

$$Q(t) = 10k + (8000 - 10k)e^{0.1t}.$$

$B(3) = 0 \Longrightarrow 0 = 10k + (8000 - 10k)e^{0.3} \Longrightarrow k = 3086.64$.

The total interest is paid

$$\begin{aligned} \int_0^3 B(t)0.1dt &= \int_0^3 10k + (8000 - 10k)e^{0.1t}dt \\ &= 3k + (8000 - k)10e^{0.1t}|_{t=0}^{t=3} \\ &= 1259.91. \end{aligned}$$

Section 2.5 2 $\frac{dy}{dt} = y(y-1)(y-2)$

Solution:

steady states: $y = 0$, $y = 1$, $y = 2$

signs of $f(y) = y(y-1)(y-2)$.

- $y < 0$, $f(y) < 0$, decreasing
- $0 < y < 1$, $f(y) > 0$, increasing
- $1 < y < 2$, $f(y) < 0$, decreasing
- $y > 2$, $f(y) > 0$, increasing

0 is unstable, 1 stable, 2 unstable.

4 $\frac{dy}{dt} = e^{-y} - 1$

Solution:

steady states: $y = 0$

signs of $f(y) = e^{-y} - 1$.

- $y < 0$, $f(y) > 0$, increasing
- $y > 0$, $f(y) < 0$, decreasing

0 is stable

6 $\frac{dy}{dt} = y^2(y^2 - 1)$

Solution:

steady states: $y = 0$, $y = 1$, $y = -1$

signs of $f(y)$.

- $y < -1$, $f(y) > 0$, increasing
- $-1 < y < 0$, $f(y) < 0$, decreasing
- $0 < y < 1$, $f(y) < 0$, decreasing
- $y > 1$, $f(y) > 0$, increasing

-1 is stable, 0 semi-stable, 1 unstable.

7 $\frac{dy}{dt} = y(1 - y^2)$

Solution:

steady states: $y = 0$, $y = 1$, and $y = -1$

signs of $f(y)$.

- $y < -1$, $f(y) > 0$, increasing
- $-1 < y < 0$, $f(y) < 0$,decreasing
- $0 < y < 1$, $f(y) > 0$, increasing
- $y > 1$, $f(y) < 0$, decreasing

-1 is stable; 0 is unstable, 1 stable.