Hw6 Solution

Section 3.1 29. Solution:

$$y'' - \frac{t(t+2)}{t^2}y' + \frac{t+2}{t^2}y = 0$$

$p(t) = -\frac{t(t+2)}{t^2}$ and

$$W = Ce^{-\int p(t)dt} = Ce^{\int \frac{t(t+2)}{t^2}dt} = Ce^{\int 1+2/tdt} = Ce^{t+2\ln|t|} = Ct^2e^{|t|}.$$

Section 3.3 5. $y'' - 2y' + 2y = 0$

Solution:

$r^2 - 2r + 2 = 0 \Longrightarrow r = \frac{2\pm\sqrt{-4}}{2} = 1 \pm i.$

We have $\lambda = 1, \quad \mu = 1$ and

$$y_1(t) = e^{\lambda t}\cos\mu t = e^t\cos t \quad y_2(t) = e^{\lambda t}\sin\mu t = e^t\sin t.$$

The general solution is $\boxed{y = C_1y_1 + C_2y_2 = C_1e^t\cos t + C_2e^t\sin t.}$

9. $y'' + 2y' + 1.25y = 0$

Solution:

$r^2 + 2r + 1.25 = 0 \Longrightarrow r = \frac{-2\pm\sqrt{-1}}{2} = -1 \pm i/2.$

We have $\lambda = -1, \quad \mu = 1/2$ and

$$y_1(t) = e^{\lambda t}\cos\mu t = e^{-t}\cos t/2 \quad y_2(t) = e^{\lambda t}\sin\mu t = e^{-t}\sin t/2.$$

The general solution is $\boxed{y = C_1y_1 + C_2y_2 = C_1e^{-t}\cos t/2 + C_2e^{-t}\sin t/2.}$

12. $y'' + 4y = 0, \quad y(0) = 0, y'(0) = 1$

Solution:

$r^2 + 4 = 0 \Longrightarrow r = \frac{\pm\sqrt{-16}}{2} = \pm 2i.$

We have $\lambda = 0, \quad \mu = 2$ and

$$y_1(t) = e^{\lambda t}\cos\mu t = \cos 2t \quad y_2(t) = e^{\lambda t}\sin\mu t = \sin 2t.$$

The general solution is $\boxed{y = C_1y_1 + C_2y_2 = C_1\cos 2t + C_2\sin 2t.}$

$y' = -2C_1 \sin 2t + 2C_2 \cos 2t.$

$$\begin{aligned} y(0) = 0 &\implies C_1 = 0 \\ y'(0) = 1 &\implies 2C_2 = 1 \\ &\implies \\ &\quad C_1 = 0 \quad C_2 = 1/2 \end{aligned}$$

$\boxed{y(t) = \frac{1}{2} \sin 2t. \text{ Steady oscillation for increasing } t.}$

13. $y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, y'(\pi/2) = 1$

Solution:

$r^2 - 2r + 5 = 0 \implies r = \frac{2 \pm \sqrt{-16}}{2} = 1 \pm 2i.$

We have $\lambda = 1, \quad \mu = 2$ and

$$y_1(t) = e^{\lambda t} \cos \mu t = e^t \cos 2t \quad y_2(t) = e^{\lambda t} \sin \mu t = e^t \sin 2t.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^t \cos 2t + C_2 e^t \sin 2t.}$

$y' = -2C_1 e^t (\cos 2t - \sin 2t) + 2C_2 e^t (\sin 2t + \cos 2t).$

$$\begin{aligned} y(\pi/2) = 0 &\implies -C_1 e^{\pi/2} = 0 \\ y'(\pi/2) = 2 &\implies -C_1 e^{\pi/2} - 2C_2 e^{\pi/2} = 2 \\ &\implies \\ &\quad C_1 = 0 \quad C_2 = -e^{-\pi/2} \end{aligned}$$

$\boxed{y(t) = e^{-\pi/2} e^2 \sin 2t. \text{ growing oscillation for increasing } t.}$

Section 3.4 1. $y'' - 2y' + y = 0$

Solution:

$r^2 - 2r + 1 = 0 \implies r_1 = r_2 = r = 1.$

We have $\lambda = 1, \quad \mu = 1$ and

$$y_1(t) = e^{rt} = e^t \quad y_2(t) = te^{rt} = te^t.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^t + C_2 te^t.}$

7. $16y'' + 24y' + 9y = 0$

Solution:

$16r^2 + 24r + 9 = 0 \Longrightarrow (4r+3)^2 = 0 \Longrightarrow r_1 = r_2 = r = -3/4.$

We have

$$y_1(t) = e^{rt} = e^{-3/4t} \quad y_2(t) = te^{rt} = te^{-3/4t}.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^{-3/4t} + C_2 t e^{-3/4t}.}$

9. $9y'' - 12y' + 4y = 0 \quad y(0) = 2, y'(0) = -1.$

Solution:

$9r^2 - 12r + 4 = 0 \Longrightarrow (3r-2)^2 = 0 \Longrightarrow r_1 = r_2 = r = 2/3.$

We have

$$y_1(t) = e^{rt} = e^{2/3t} \quad y_2(t) = te^{rt} = te^{2/3t}.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^{2/3t} + C_2 t e^{2/3t}.}$

Therefore

$$y'(t) = 2/3C_1 e^{2/3t} + C_2 e^{2/3t} + 2/3 C_2 t e^{2/3t}.$$

$$\begin{aligned} y(0) = 2 \quad &\Longrightarrow \quad C_1 = 2 \\ y'(0) = -1 \quad &\Longrightarrow \quad 2/3C_1 + C_2 = -1 \\ &\Longrightarrow \\ & \quad\quad C_1 = 2 \quad C_2 = -7/3. \end{aligned}$$

$\boxed{y(t) = 2e^{2/3t} - 7/3te^{2/3t} \quad y \to -\infty \text{ as } t \to +\infty.}$

12. $y'' - y' + y/4 = 0 \quad y(0) = 2, y'(0) = b.$

Solution:

$r^2 - r + 1/4 = 0 \Longrightarrow (r - 1/2)^2 = 0 \Longrightarrow r_1 = r_2 = r = 1/2.$

We have

$$y_1(t) = e^{rt} = e^{t/2} \quad y_2(t) = te^{rt} = te^{t/2}.$$

The general solution is $\boxed{y = C_1 y_1 + C_2 y_2 = C_1 e^{t/2} + C_2 t e^{t/2}.}$

Therefore

$$y'(t) = C_1/2e^{t/2} + C_2 e^{t/2} + C_2/2te^{t/2}.$$

$$
\begin{aligned}
y(0) = 2 &\implies C_1 = 2 \\
y'(0) = b &\implies C_1/2 + C_2/2 = b \\
&\implies \\
& \quad C_1 = 2 \quad C_2 = 2b - 2.
\end{aligned}
$$

$$\boxed{y(t) = 2e^{t/2} + (2b-2)te^{t/2} = e^{t/2}\left(2 + (2b-2)t\right)}$$

When $2b - 2 = 0 \implies b = 1$, $y = 2e^{t/2}$ is always positive. When $b \neq 1$, y can be negative depending on the value of t. Therefore $b = 1$ that seperates solutions that beome negative from those that are always positive.