Hw8

1 Determine ω_0, R, and δ so as to write the given expression in the form $u = R\cos(\omega_0 t - \delta)$.

(1) $u = 3\cos(2t) + 4\sin(2t)$;

(2) $u = -3\cos(2t) + 4\sin(2t)$.

Solution:

(1)

$$\begin{aligned} u &= 3\cos 2t + 4\sin 2t \\ &= \sqrt{3^2+4^2}\cos(2t-\delta) \\ &= 5\cos(2t-\delta), \end{aligned}$$

where

$$\tan\delta = \frac{4}{3} \Longrightarrow \delta \approx 0.9273.$$

Therefore, $\omega_0 = 2$, $R = 5$, and $\delta \approx 0.9273$.

(2)

$$\begin{aligned} u &= -3\cos 2t + 4\sin 2t \\ &= \sqrt{3^2+4^2}\cos(2t-\delta) \\ &= 5\cos(2t-\delta), \end{aligned}$$

where

$$\tan\delta = -\frac{4}{3} \Longrightarrow \delta \approx \pi - 0.9273 = 2.2143.$$

Therefore, $\omega_0 = 2$, $R = 5$, and $\delta \approx 2.2143$.

2. A mass weighting 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in and then released, and if there is no danping, determine the position u of the mass at any time t, Find the frequency, period, amplitude, and phase of the motion.

Solution:

$w = 2,\ mg = w \Longrightarrow m = \frac{w}{g} = \frac{2}{32}$.

$kL = mg \Longrightarrow k = \frac{mg}{L} = \frac{2}{6/12} = 4$.

We have $mu'' + ku = 0 \Longrightarrow \frac{2}{32}u'' + 4u = 0$ with $u(0) = 3/12 = 1/4$ (the mass pulled down 3 inch) and $u'(0) = 0$ (release without any velocity).

We solve this initial value problem as follows:

$2/32r^2 + 4 = 0 \Longrightarrow r = \pm 8i$.

We have $\lambda = 0, \quad \mu = 8$ and

$$u_1(t) = e^{\lambda t}\cos\mu t = \cos 8t \quad u_2(t) = e^{\lambda t}\sin\mu t = \sin 8t.$$

The general solution is $\boxed{u = C_1u_1 + C_2u_2 = C_1\cos 8t + C_2\sin 8t.}$

$u' = -8C_1\sin 8t + 8C_2\cos 8t$.

$$\begin{aligned} u(0) = 1/4 &\Longrightarrow C_1 = 1/4 \\ u'(0) = 0 &\Longrightarrow C_2 = 0 \\ &\Longrightarrow \\ & \quad C_1 = 1/4 \quad C_2 = 0 \end{aligned}$$

Therefore the solution is $\boxed{u(t) = 1/4\cos 8t.}$

$\omega = 8$, $T = \frac{2\pi}{\omega} = \pi/4$, $R = \frac{1}{4}$, $\delta = 0$.