Math 781 Hw1

due Monday 08/29.

1. Choose the correct assertions (in each, $n \to \infty$)

   a. $(n+1)/n^2 = o(1/n)$

   b. $(n+1)/\sqrt{n} = o(1)$

   c. $1/\ln n = O(1/n)$

   d. $1/(n \ln n) = o(1/n)$

   e. $e^n/n^5 = O(1/n)$

2. Show that these assertions are not true.

   a. $e^x - 1 = O(x^2)$ as $x \to 0$

   b. $x^{-2} = O(\cot x)$ as $x \to 0$

   c. $\cot x = o(x^{-1})$ as $x \to 0$

3. Show that if $x_n = O(\alpha_n)$, then $x_n/\ln n = o(\alpha_n)$.

4. Define a sequence by

$$x_0 = 2; \quad x_{n+1} = \frac{1}{2}x_n + \frac{1}{x_n} \quad \forall\ n > 0.$$

   It is known that $x_n \to \sqrt{2}$. Prove that the order of the convergence is quadratic.

5. Convert $x = 12.5$ into a binary expression with 5 digits.