Math 781 Hw10

due Monday 11/07/2022.

1. Derive the formula for approximating the derivative.

$$f'(x) \approx \frac{1}{2h}\left(-3f(x) + 4f(x+h) - f(x+2h)\right).$$

2. Using Taylor series to derive the error term for the formula in Problem 1.

3. Suppose that $N(h)$ is an approximation to M for every $h > 0$ and

$$M - N(h) = K_1 h + K_2 h^2 + K_3 h^3 + \cdots,$$

where K_1, K_2, K_3 are nonzeros constants independent of h. Use $N(h)$, $N(h/2)$, and $N(h/3)$ to produce an $O(h^3)$ approximation to M.

4. Derive a numerical differentiation formula of order $O(h^4)$ by applying Richardson extrapolation to

$$f'(x) = \frac{1}{2h}\left(f(x+h) - f(x-h)\right) - \frac{h^2}{6}f'''(x) - \frac{h^4}{120}f^{(5)}(x) + \cdots$$

Give the error term of order $O(h^4)$.