Math 781 Hw11
due Monday 11/14/2022.

1. Verify the following formula is exact for polynomials of degree ≤ 4.

$$\int_0^1 f(x)dx \approx \frac{1}{90}\left[7f(0) + 32f\left(\frac{1}{4}\right) + 12f\left(\frac{1}{2}\right) + 32f\left(\frac{3}{4}\right) + 7f(1)\right].$$

2. Find the formula

$$\int_0^1 f(x)dx \approx A_0 f(0) + A_1 f(1)$$

that is exact for all functions of the form $f(x) = ae^x + b\cos(\pi x/2)$.

3. Use the Lagrange interpolation polynomial to derive the formula of the form

$$\int_0^1 f(x)dx \approx Af\left(\frac{1}{3}\right) + Bf\left(\frac{2}{3}\right).$$

Transform this formula to one for integration over $[a, b]$.

4. Determine values for A, B, C that make the formula

$$\int_0^2 xf(x)dx \approx Af(0) + Bf(1) + Cf(2)$$

exact for all polynomials of degree as high as possible. What is the maximum degree?