

Math 781 Hw3
due Monday 09/12/2022.

1. How many bits of precision are lost in a computer when we carry out the subtraction $x - \sin x$ for $x = \frac{1}{2}$?
2. Suggest ways to avoid loss of significance in these calculations.

(a) $\sqrt{x^2 + 1} - x$

(b) $x^{-3}(\sin x - x)$

(c) $\sqrt{x + 2} - \sqrt{x}$

3. Find analytically the solution of this difference equation with the given initial values:

$$\begin{cases} x_0 = 1, & x_1 = 0.9 \\ x_{n+1} = -0.2x_n + 0.99x_{n-1}. \end{cases}$$

Without computing the solution recursively, predict whether such a computation would be stable.

4. What are the condition numbers of the following functions? Where are they large?

(1) $f(x) = (x - 1)^\alpha$; (2) $\ln x$; (3) $\sin x$.