

**Math 781 Hw5**  
due Monday 09/26/2022.

1. Perform four iterations of Newton's method for the polynomial

$$f(x) = 4x^3 - 2x^2 + 3$$

starting with  $x_0 = -1$ .

2. Devise a Newton's algorithm for computing the fifth root of any positive number.
3. Suppose that  $p$  is a double zero of the function  $f$ . Thus  $f(p) = f'(p) = 0 \neq f''(p)$ . Show that if  $f''$  is continuous, then in Newton's method we shall have

$$\lim_{n \rightarrow \infty} \frac{|p - p_n|}{|p - p_{n-1}|} = \frac{1}{2}.$$

4. Suppose  $f(x) = (x - p)^k h(x)$ , where  $k \geq 1$  is an integer,  $h(p) \neq 0$  and  $h'''(x)$  is continuous in a neighborhood of  $p$ . Prove the modified Newton's method

$$p_n = p_{n-1} - \frac{k f(p_{n-1})}{f'(p_{n-1})}$$

converges at least quadratically. (Hint: Use the fixed point iteration result to show  $\lim_{x \rightarrow p} g'(x) = 0$ .)