Math 781 Hw7

due Monday 10/17/2022.

1. Prove that if f is a polynomial degree k, then for $n > k$,

$$f[x_0, x_1, \cdots, x_n] = 0.$$

2. Let $L_i(x)$ be the Lagrange interpolation polynomial basis which interpolates a function at $x_0, \cdots x_n$. Show for any f,

$$\sum_{i=0}^{n} f(x_i)L_i(x) = \sum_{i=0}^{n} f[x_0, \cdots, x_i]\Pi_{j=0}^{i-1}(x - x_j).$$

   Use this result to show

$$f[x_0, \cdots, x_n] = \sum_{i=0}^{n} f(x_i)\Pi_{j=0, j\neq i}^{n}(x_i - x_j)^{-1}.$$

3. Determine the Newton interpolation polynomial for this table:

x	0	1	2	7
y	51	3	1	201

4. Obtain a formula for the polynomial p of least degree that takes these values:

$$p(x_i) = y_i, \quad p'(x_i) = 0, \quad , i = 0, \cdots, n.$$

   (Hint: use the Hermite interpolation polynomials).