Math 781 Hw9

due Monday 10/31/2022.

1. Determine the degree 5 Chebyshev polynomial $T_5(x)$ using the three-term recurrence.

2. Find the linear least squares approximation to $f(x) = \sin(x)$ on the interval $[-1, 1]$.

3. Let $w(x) = \frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$. Show that the Chebyshev polynomials $T_n(x) = \cos(n \arccos x)$ $(n \geq 0)$ satisfy

$$\int_{-1}^{1} w(x) T_i(x) T_j(x)\, dx = 0, \quad \forall i \neq j,$$

   and dtermine $\int_{-1}^{1} w(x) \left(T_n(x)\right)^2\, dx$.

   Hint: Use substitution with $x = \cos\theta$ with $\theta \in [0, \pi]$.