## Math 782 Hw1

due Tuesday 01/23/2018.

1. Is it true that for $A \in \mathbb{C}^{m \times n}$ and $B \in \mathbb{C}^{n \times p}, A B=0$ implies either $A=0$ or $B=0$ ? Prove or give a counter-example.
2. Prove that $A \in \mathbb{C}^{m \times n}$ is rank 1 if and only if there exist nonzero vectors $u \in \mathbb{C}^{m}$ and $v \in \mathbb{C}^{n}$ such that $A=u v^{*}$.
3. Suppose $A \in \mathbb{C}^{m \times m}$ is nonsingular and $u, v \in \mathbb{C}^{m}$.
(a) Prove that if $B=A+u v^{*}$ is nonsingular, then its inverse has the form

$$
B^{-1}=A^{-1}+\alpha A^{-1} u v^{*} A^{-1}
$$

for some scalar $\alpha$. Determine the value of $\alpha$. Hint: Multiply out $B B^{-1}$ and set it to I. This is the Sherman-Morrison formula.
(b) Find condition(s) on $A, u$, and $v$ such that $B$ is singular.

