## Math 782 Hw1

## due Tuesday 01/23/2018.

- 1. Is it true that for  $A \in \mathbb{C}^{m \times n}$  and  $B \in \mathbb{C}^{n \times p}$ , AB = 0 implies either A = 0 or B = 0? Prove or give a counter-example.
- 2. Prove that  $A \in \mathbb{C}^{m \times n}$  is rank 1 if and only if there exist nonzero vectors  $u \in \mathbb{C}^m$  and  $v \in \mathbb{C}^n$  such that  $A = uv^*$ .
- 3. Suppose  $A \in \mathbb{C}^{m \times m}$  is nonsingular and  $u, v \in \mathbb{C}^m$ .
  - (a) Prove that if  $B = A + uv^*$  is nonsingular, then its inverse has the form

$$B^{-1} = A^{-1} + \alpha A^{-1} u v^* A^{-1}$$

for some scalar  $\alpha$ . Determine the value of  $\alpha$ . Hint: Multiply out  $BB^{-1}$  and set it to I. This is the Sherman-Morrison formula.

(b) Find condition(s) on A, u, and v such that B is singular.