Math 782 Hw10

due Tuesday 04/24/2018

1. Suppose $A = QTQ^*$ is a Schur form of A, where T is upper triangular and Q is unitary.
   (a) Show for any $\mu \in \mathbb{C}$, $A - \mu I = Q(T - \mu I)Q^*$ is a Schur form of $A - \mu I$.
   (b) If A is invertible, show $A^{-1} = QT^{-1}Q^*$ is a Schur form of A^{-1}.

2. Let
$$A = \begin{bmatrix} 8 & 1 & 0 \\ 1 & 4 & \epsilon \\ 0 & \epsilon & 1 \end{bmatrix}, \quad \epsilon \in \mathbb{R}, \quad |\epsilon| < 1.$$
   (a) Show that A has only real eigenvalues for any ϵ with the given conditions.
   (b) Use Gershgorin's theorem to estimate the eigenvalues of A.
   (c) Find a way to establish the tighter bound $|\lambda_3 - 1| \le \epsilon^2$ on the smallest eigenvalue of A.
   Hint: Consider diagonal similarity transformation with diagonal($1,\ 1,\ \frac{1}{\epsilon}$*).*

3. Supose $A, E \in \mathbb{C}^{m\times m}$ and A is normal. Use the Bauer-Fike Theorem to prove that for any eigenvalue $\tilde{\lambda}$ of $\widetilde{A} = A + E$, there exists an eigenvalue λ_i of A such that
$$|\tilde{\lambda} - \lambda_i| \le \|E\|_2.$$