## Math 782 Hw3

due Tuesday 02/06/2018

1. Determine a compact SVD for the rank-one matrix  $A = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \end{bmatrix}$ .

- 2. Consider the matrix  $A = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix}$ .
  - (a) Determine a real full SVD of A in the form  $A = U\Sigma V^T$ . Hint: Follow the proof. Find the eigenvalues and eigenvectors of  $A^T A$  to determine  $\Sigma$  and V. Then determine U.
  - (b) List the singular values, left singular vectors, and the right singular vectors.
  - (c) Use the full SVD of A to determine a full SVD for  $A^T$  and  $A^{-1}$ .
- 3. Let  $A \in \mathbb{C}^{m \times n}$ . Prove that

$$||A||_2 = \max_{x \in \mathbb{C}^n, \ y \in \mathbb{C}^m, \ ||x||_2 = 1, \ ||y||_2 = 1} |y^* A x|.$$

Hint: Use the SVD of A to prove  $\max_{x \in \mathbb{C}^n, y \in \mathbb{C}^m, \|x\|_2 = 1, \|y\|_2 = 1} |y^*Ax| = \sigma_1 = \max_i \sigma_i$ .