## Math 782 Hw3

## due Tuesday 02/06/2018

1. Determine a compact SVD for the rank-one matrix $A=\left[\begin{array}{c}1 \\ -2 \\ 2\end{array}\right]\left[\begin{array}{ll}3 & 4\end{array}\right]$.
2. Consider the matrix $A=\left[\begin{array}{cc}-2 & 11 \\ -10 & 5\end{array}\right]$.
(a) Determine a real full SVD of $A$ in the form $A=U \Sigma V^{T}$.

Hint: Follow the proof. Find the eigenvalues and eigenvectors of $A^{T} A$ to determine $\Sigma$ and $V$. Then determine $U$.
(b) List the singular values, left singular vectors, and the right singular vectors.
(c) Use the full SVD of $A$ to determine a full SVD for $A^{T}$ and $A^{-1}$.
3. Let $A \in \mathbb{C}^{m \times n}$. Prove that

$$
\|A\|_{2}=\max _{x \in \mathbb{C}^{n}, y \in \mathbb{C}^{m},\|x\|_{2}=1,\|y\|_{2}=1}\left|y^{*} A x\right| .
$$

Hint: Use the SVD of $A$ to prove $\max _{x \in \mathbb{C}^{n}, y \in \mathbb{C}^{m},\|x\|_{2}=1,\|y\|_{2}=1}\left|y^{*} A x\right|=\sigma_{1}=\max _{i} \sigma_{i}$.

