Math 782 Hw7

due Tuesday 03/13/2018

1. Supose $A = [a_{ij}]$, $E = [e_{ij}] \in \mathbb{C}^{m\times n}$ and $\widetilde{A} := A + E$ with the elements satisfying

$$\frac{|e_{ij}|}{|a_{ij}|} \leq c\epsilon_{\text{machine}},$$

for $i = 1, \cdots, m$ and $j = 1, \cdots, n$, where c is a constant. If $a_{ij} = 0$, we interpret that the inequality implies $e_{ij} = 0$.

Derive an upper bound for

$$\frac{\|\widetilde{A} - A\|}{\|A\|}$$

where $\|\cdot\|$ is (a): $\|\cdot\|_1$, (b): $\|\cdot\|_\infty$, (c): $\|\cdot\|_F$.

Hint: For each norm, use its definition to derive an upper bound for $\|\widetilde{A} - A\| = \|E\|$.

2. Assume that x, y, and z are floating point numbers. Show that the algorithm of computing $f(x, y, z) = z - xy$ is backward stable in a finite floating point number system with machine precision $\epsilon_{\text{machine}}$.