Math 782 Hw8

due Tuesday 04/03/2018

1. Suppose $C \in \mathbb{C}^{m\times m}$ and $x \in \mathbb{C}^m$. Consider the matrix-vector multiplication $b = Cx$. On a finite floating point number system with machine precision $\epsilon_{\text{machine}}$, the computed approximation to b is $\tilde{b}$, which satisfies

$$\tilde{b} = (C + \delta C)x, \quad \frac{\|\delta C\|_2}{\|C\|_2} = O(\epsilon_{\text{machine}}).$$

Here $a = O(\epsilon_{\text{machine}})$ means $a = c\epsilon_{\text{machine}}$ for some moderate constant $c \geq 0$.

Now, suppose C is unitary. Prove

(a)

$$\tilde{b} = C(x + \delta x)$$

for some vector $\delta x \in \mathbb{C}^m$ satisfying $\frac{\|\delta x\|_2}{\|x\|_2} = O(\epsilon_{\text{machine}})$.

(b)

$$\tilde{b} = b + \delta b$$

for some vector $\delta b \in \mathbb{C}^m$ satisfying $\frac{\|\delta b\|_2}{\|b\|_2} = O(\epsilon_{\text{machine}})$.

2. Consider the example

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1.001 \\ 1 & 1.001 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 0.001 \\ 4.001 \end{bmatrix}.$$

(a) Compute the matrices A^+ and P, the orthogonal projector onto range(A). Give the exact answers.

Hint: You may use the formulas $A^+ = (A^TA)^{-1}A^T$ and $P = A(A^TA)^{-1}A^T$, and use the inverse formula for 2×2 matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

(b) Find the exact solution x to the least squares problem $\min_x \|b - Ax\|_2$.

(c) Compute $\kappa_2(A)$, $\theta = cos^{-1}\left(\frac{\|Pb\|_2}{\|b\|_2}\right)$, and $\eta = \frac{\|A\|_2\|x\|_2}{\|Ax\|_2}$.

(d) Compute $\frac{(\kappa_2(A))^2 \tan(\theta)}{\eta}$ and $\frac{\kappa_2(A)}{\eta\cos\theta}$.

For Part (c) and (d), you can use numerical answers.