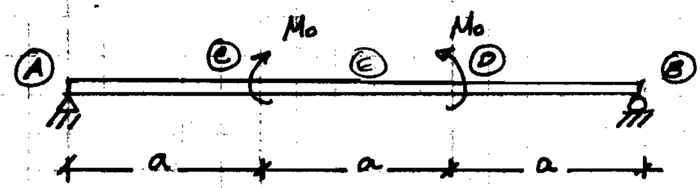
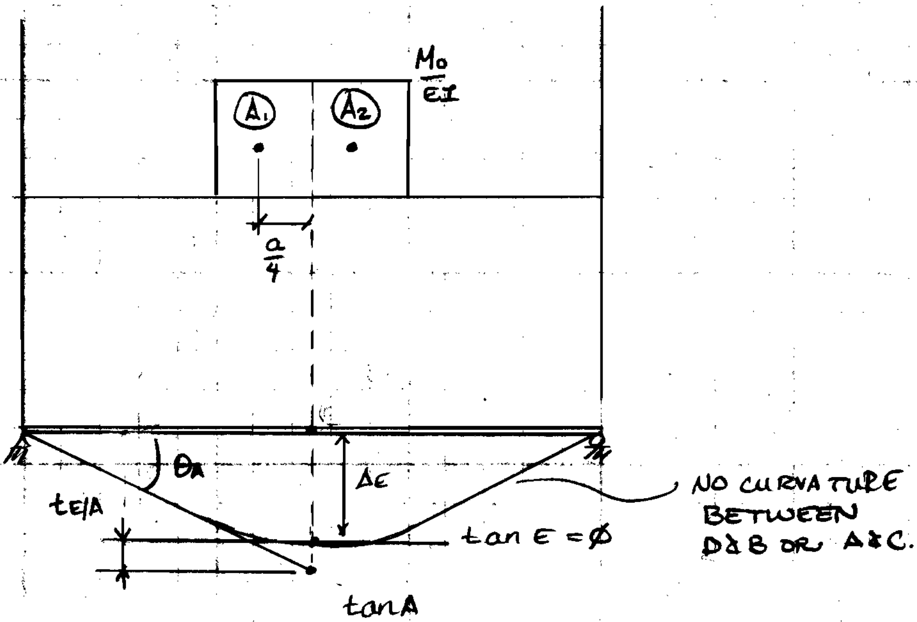
8.6*) Re-do problem 8-6, this time using moment area theorems to solve for maximum deflection. EI is constant.



CURVATURE DIAGRAM: $\dfrac{M(x)}{EI}$



$$\Delta max = \Delta_E$$

$$\Delta_E = \theta_A\left(a+\frac{a}{2}\right) + t_{E/A}$$

$$t_{E/A} = A_1\left(\frac{a}{4}\right) = \frac{M_o}{EI}\left(\frac{a}{2}\right)\left(\frac{a}{4}\right) = \frac{M_o a^2}{8EI}$$

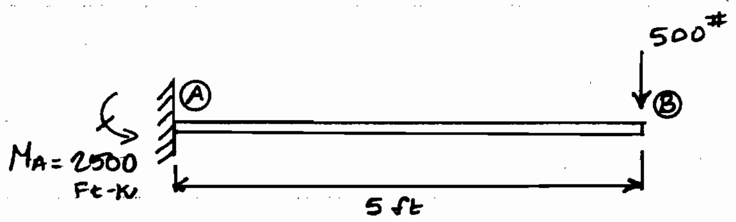
$$\theta_A = \theta_{E/A} = \theta_E - \theta_A = -A_1 = -\frac{M_o}{EI}\left(\frac{a}{2}\right) = (-)\frac{M_o a}{2EI}$$

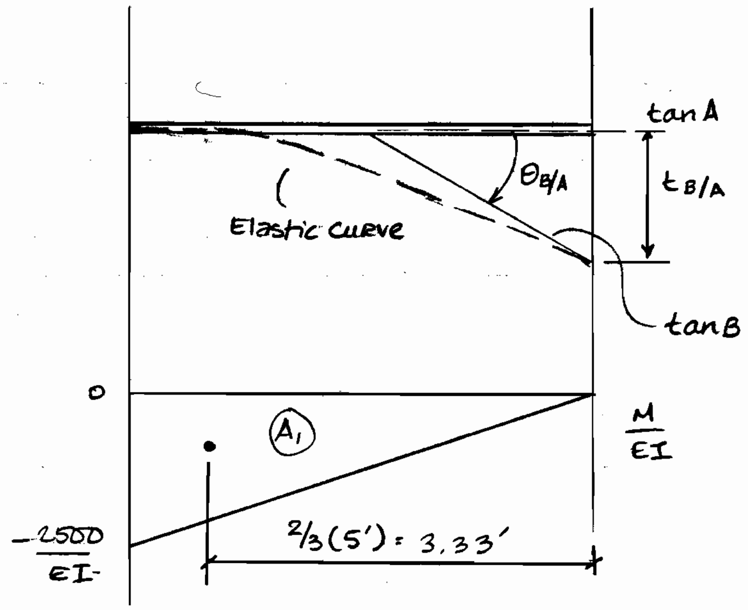
$$\boxed{\theta_A = (-)\frac{M_o a}{2EI}, \text{ neg} \therefore \circlearrowright}$$

$$\Delta_E = \left[(-)\frac{M_o a}{2EI}\left(\frac{3a}{2}\right) + \frac{M_o a^2}{8EI}\right] = \left[(-)\frac{3M_o a^2}{4EI} + \frac{M_o a^2}{8EI}\right] = -\frac{5}{8}\frac{M_o a^2}{EI}$$

$$\boxed{\Delta max = -\frac{5M_o a^2}{8EI}, \text{ neg} \therefore \downarrow}$$

8-10) Use moment area theorems & determine slope & deflection at B. EI is constant.





$$A_1 = \frac{1}{2}\left(-\frac{2500}{EI}\right)(5') = \frac{6250}{EI}$$

$$\theta_{B/A} = \theta_B + \cancel{\theta_A}^{0}$$

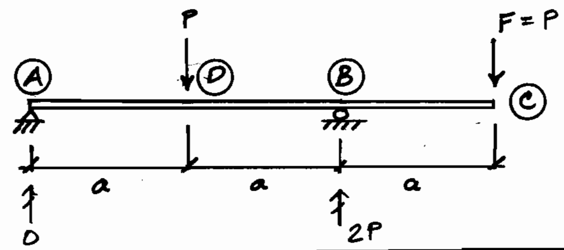
$$\theta_{B/A} = \theta_B = A_1 = \frac{6250}{EI}$$

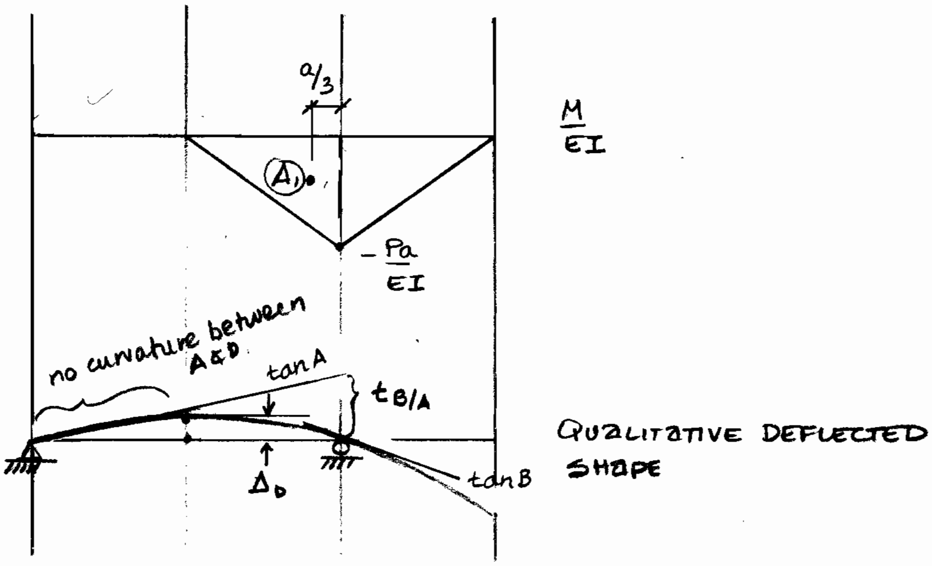
$$\boxed{\theta_B = \frac{6250}{EI}}$$

$$\Delta_B = |t_{B/A}| = A_1\bar{X} = \frac{6250}{EI}(3.33') = \frac{20,833.33}{EI}$$

$$\boxed{\Delta_B = \frac{20,833.33}{EI}}$$

Prob. 8-15) The beam is subjected to the load P as shown. If $F = P$, determine the displacement at D. Use the moment-area theorems. EI is constant.





$$A_1 = a\left(-\frac{Pa}{EI}\right)\left(\tfrac{1}{2}\right) = -\frac{Pa^2}{2EI}$$

$$t_{B/A} = A_1\bar{X} = -\frac{Pa^2}{2EI}\left(\frac{a}{3}\right) = -\frac{Pa^3}{6EI}$$

$t_{D/A} = 0$ (no area under M/EI diagram between A & D)

$$\Delta_D = \frac{1}{2}\left|t_{B/A}\right| - \left|t_{D/A}\right|$$

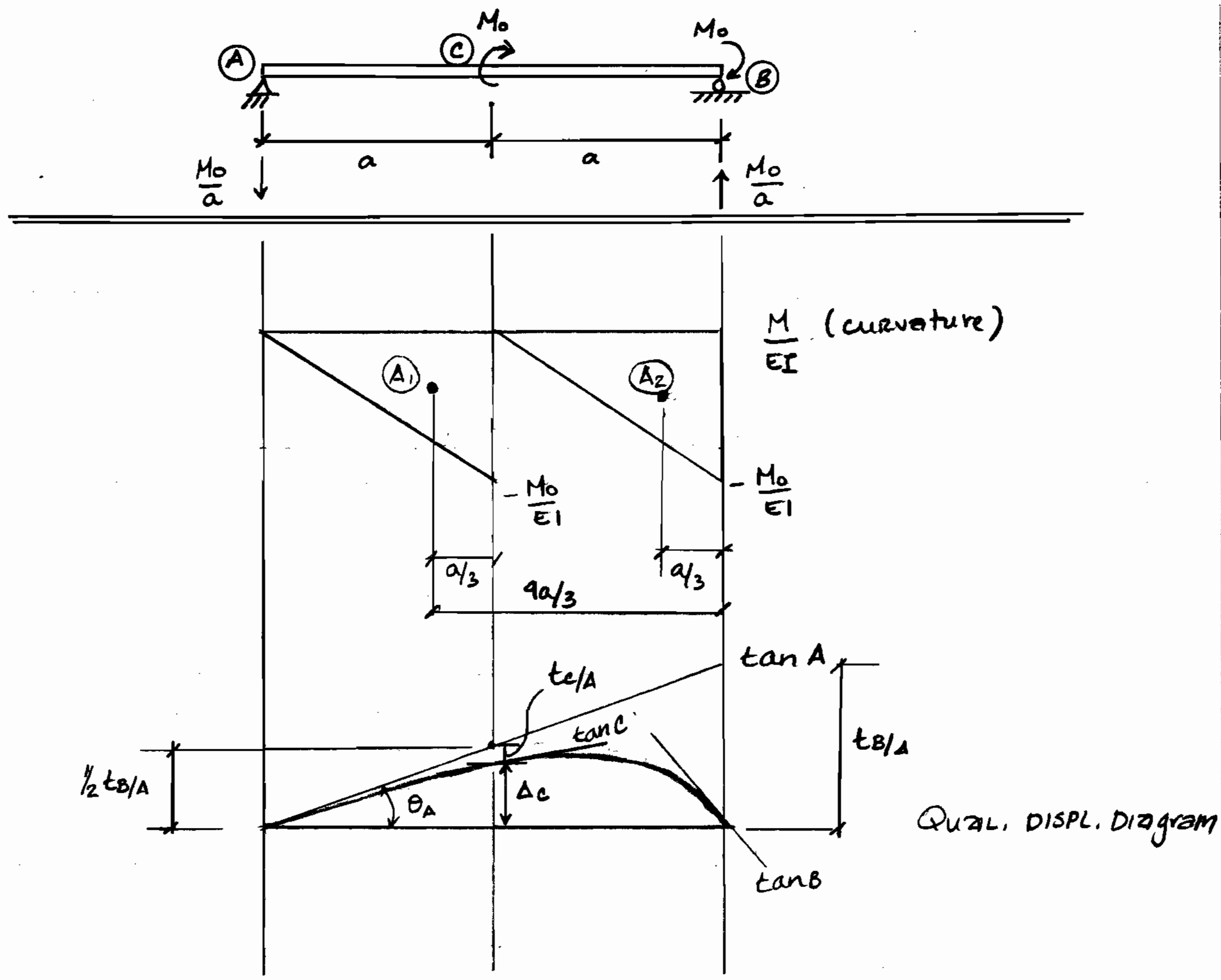
$$= \frac{Pa^3}{12EI} - 0$$

$$= \frac{Pa}{12EI}, \text{ pos} \therefore \uparrow$$

$$\boxed{\Delta_D = \frac{Pa}{12EI} \uparrow}$$

Problem No. 8-19 | Assignment No. 6 | C.R. Bennett 03-08-06

8-19) The shaft is subjected to the loading shown. If the bearings at A & B only exert vertical reactions on the shaft, determine the slope at A and the displacement at C. Use the moment-area theorems. EI is constant.



$$A_2 = A_1 = \frac{1}{2}(a)\left(-\frac{M_o}{EI}\right) = -\frac{M_o a}{2EI}$$

$$t_{B/A} = A_2\left(a/3\right) + A_1\left(4a/3\right)$$

$$= -\frac{M_o a}{2EI}\left(\frac{a}{3}\right) - \frac{M_o a}{2EI}\left(\frac{4a}{3}\right)$$

$$= -\frac{M_o a^2}{6EI} - \frac{4M_o a^2}{6EI}$$

$$= -\frac{5M_o a^2}{6EI}$$

$$t_{C/A} = A_1\left(a/3\right)$$

$$= -\frac{M_0 a}{2EI}\left(\frac{a}{3}\right) = -\frac{M_0 a^2}{6EI}$$

Slope at A:

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{5M_0 a^2}{6EI(2a)} = \frac{5M_0 a}{12EI}$$

$$\boxed{\theta_A = \frac{5M_0 a}{12EI}}$$ ← SLOPE AT A

DISPLACEMENT AT C

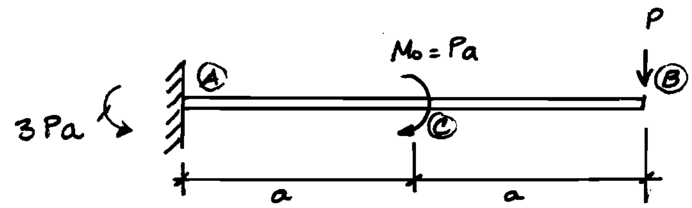
$$\Delta_C = \left|\frac{1}{2}t_{B/A}\right| - \left|t_{C/A}\right|$$

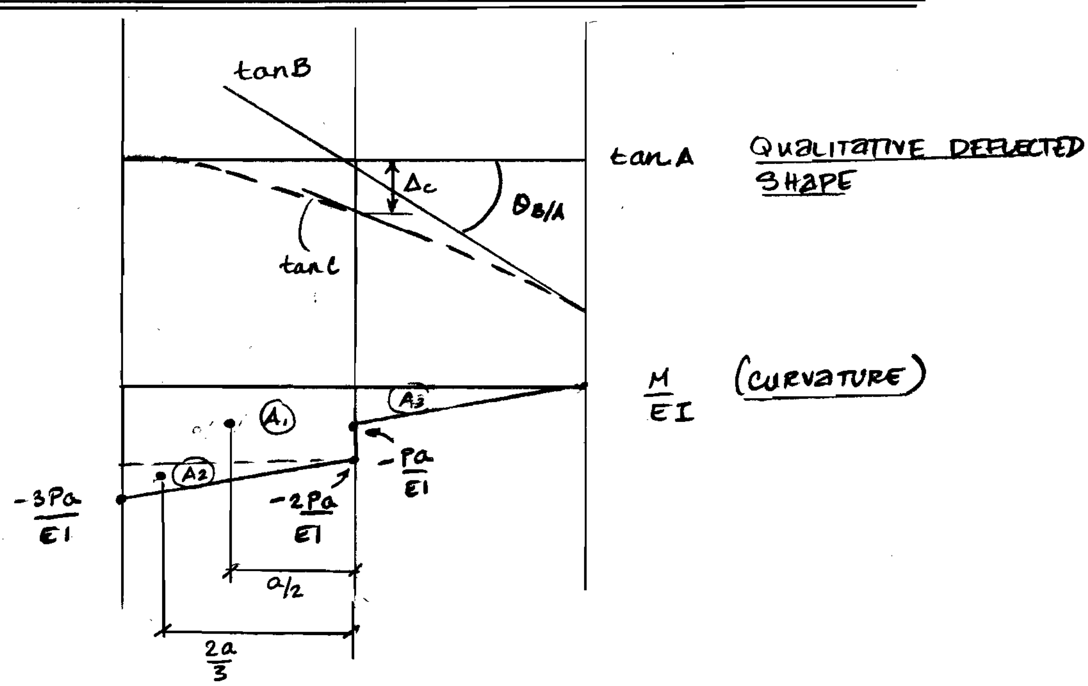
$$= \frac{1}{2}\left(\frac{5M_0 a^2}{6EI}\right) - \left(\frac{M_0 a^2}{6EI}\right)$$

$$= \frac{M_0 a^2}{4EI}$$

$$\boxed{\Delta_C = \frac{M_0 a^2}{4EI}}$$ ← DISPLACEMENT AT C.

8-21) Use the moment-area theorems and determine the slope at B and the deflection at C. EI is constant.





Slope at B

$$\theta_B = \theta_{B/A} + \cancel{\theta_A}^{\,0} = \tfrac{1}{2}\left(-\frac{Pa}{EI}\right)(a) + \tfrac{1}{2}\left[-\frac{3Pa}{EI} - \frac{2Pa}{EI}\right](a)$$

$$\boxed{\theta_B = \frac{3Pa^2}{EI}}$$

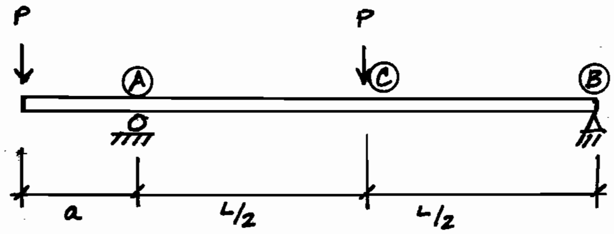
Deflection at C

$$\Delta_c = t_{C/A} = A_1\left(\frac{a}{2}\right) + A_2\left(\frac{2a}{3}\right)$$

$$\Delta_c = \left[\left(-\frac{2Pa}{EI}\right)(a)\cdot\left(\frac{a}{2}\right)\right] + \left[\frac{1}{2}(a)\left(-\frac{Pa}{EI}\right)\left(\frac{2a}{3}\right)\right]$$

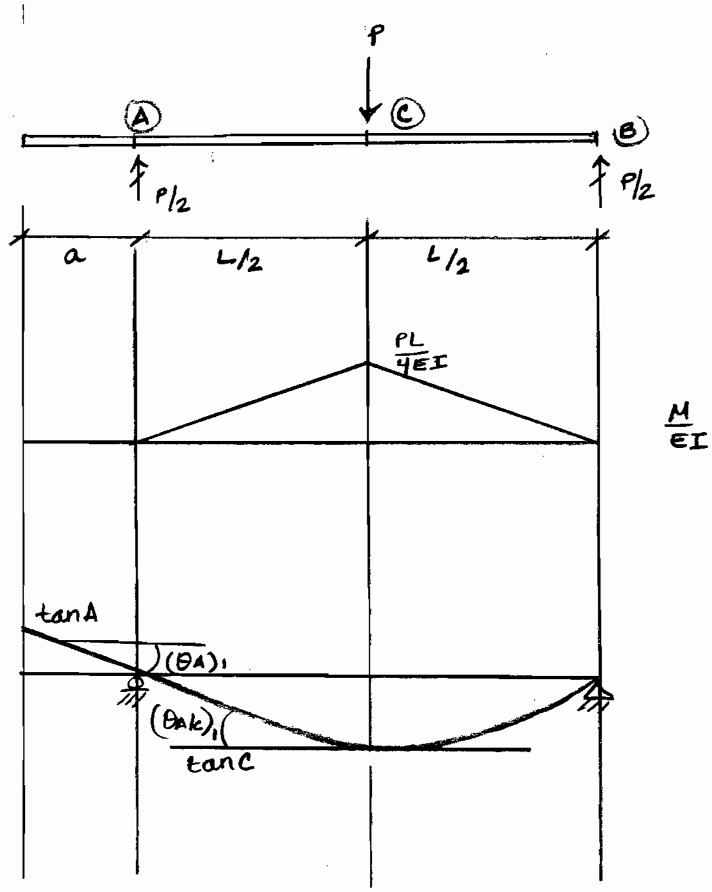
$$\boxed{\Delta_c = \frac{4Pa^3}{3EI}}$$

8-23) Use the moment-area theorems & determine the value of a so that the slope at A is equal to zero. EI is constant.



Consider the cases if each "P" was applied separately, & solve for θ_A.

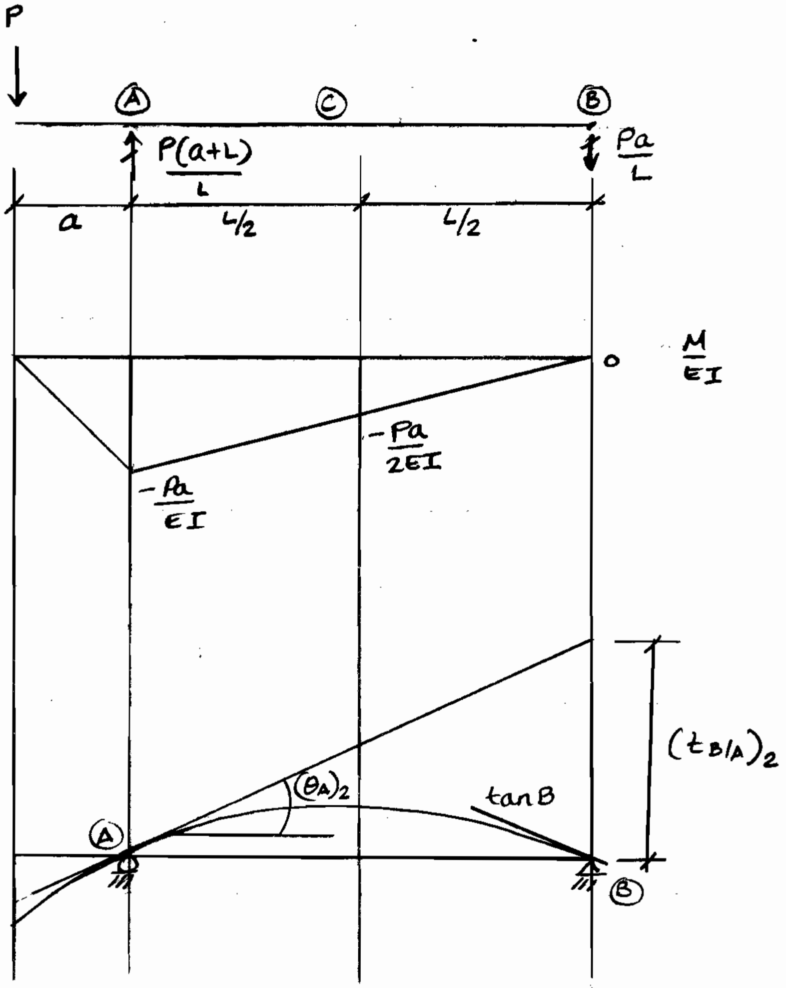
CASE I



$$(\theta_A)_1 = (\theta_{A/C})_1 = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)$$

$$(\theta_A)_1 = \frac{PL^2}{16EI}$$

CASE II



$$(t_{B/A})_2 = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$(\theta_A)_2 = \frac{|(t_{B/A})_2|}{L} = \frac{PaL}{3EI}$$

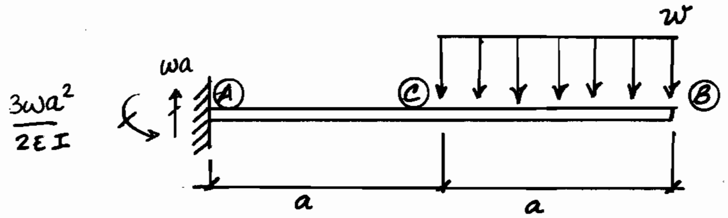
Solve for "a" such that the slope at A is zero under applied loads.

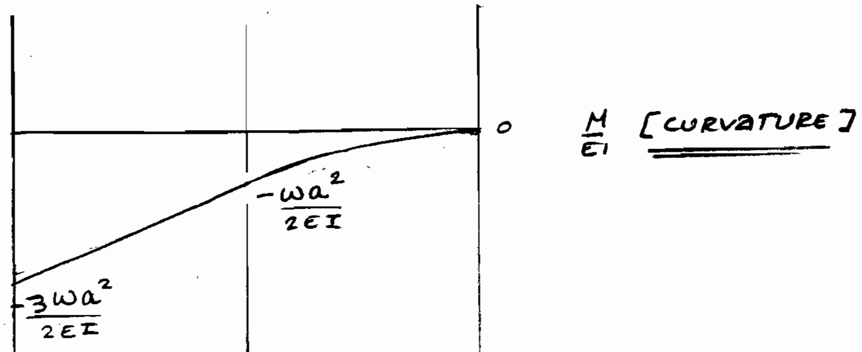
Require that $\theta_A = 0 = (\theta_A)_1 - (\theta_A)_2$

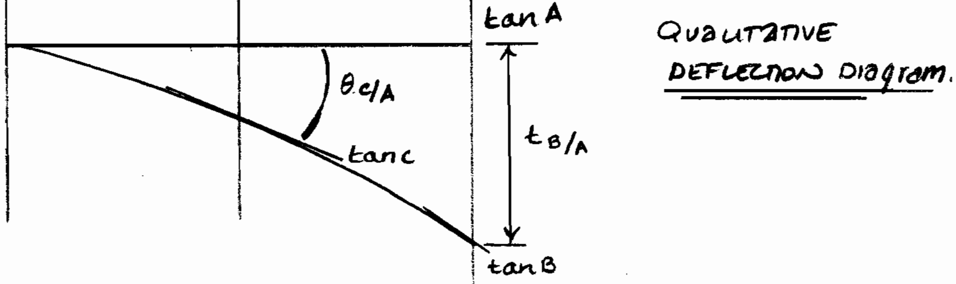
$$0 = \frac{PL^2}{16EI} - \frac{PaL}{3EI}$$

$$\boxed{a = \frac{3}{16}L}$$

8-25) Use the moment-area theorems & determine the slope at C and displacement at B. EI is constant.







SLOPE AT C

$$\theta_C = |\theta_{C/A}| = \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a) + \left(-\frac{wa^2}{2EI}\right)(a)$$

$$\boxed{\theta_C = \frac{wa^3}{EI}}$$

DEFLECTION AT B

$$\Delta_B = |t_{B/A}| = \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a)\left(a + \frac{2}{3}a\right) + \left(-\frac{wa^2}{2EI}\right)(a)\left(a + \frac{a}{2}\right) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(\frac{3}{4}a\right)$$

$$\boxed{\Delta_B = \frac{41wa^4}{24EI}}$$