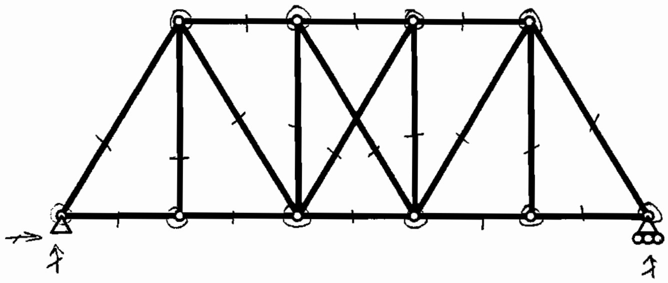
CE 461 – Structural Analysis, Fall 2006 **Name:** C. R. Bennett
EXAM No. 1

1. [16 pts.] Label the following structures as determinate, indeterminate, or unstable. If the structure is indeterminate, indicate the degree of indeterminacy.



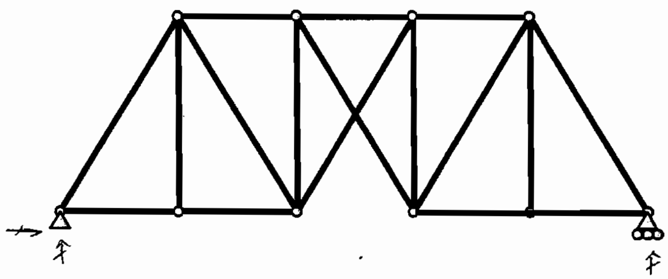
(a) $b=18$ $j=10$ $r=3$

$b+r \stackrel{?}{=} 2j$

$18+3 = 2(10)$

$21 \geq 20$ ∴ 1° ind. iff stable

Stable ∴ 1° indeterminate



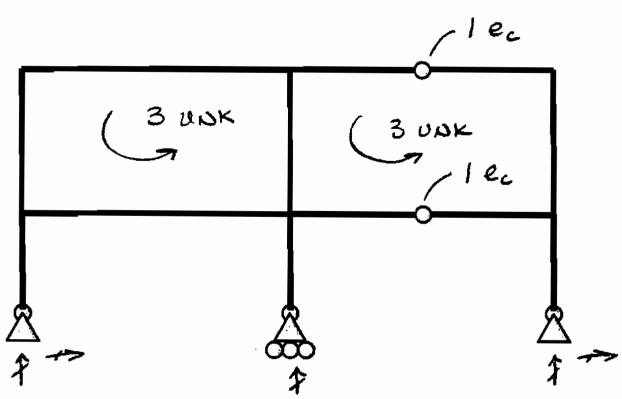
(b) $b=17$ $j=10$ $r=3$

$b+r \stackrel{?}{=} 2j$

$17+3 = 2(10)$

$20 = 20$ ∴ det. iff stable

Stable ∴ determinate

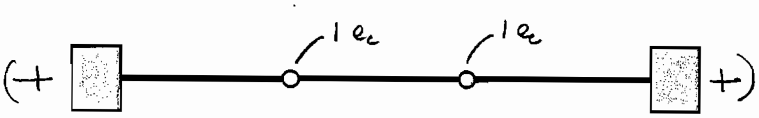


(c) UNK vs. EQNS

$5+3+3$ vs. $3+1+1$

$11 > 5$ ∴ 6° ind. iff stable

Stable ∴ 6° indeterminate

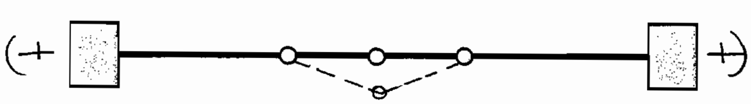


(d) UNK vs. EQNS

6 vs. $3+2$

$6 > 5$ ∴ 1° ind. iff stable

Stable ∴ 1° indet.

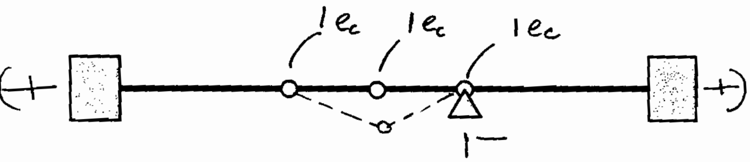


(e) UNK vs. EQNS

6 vs. $3+3$

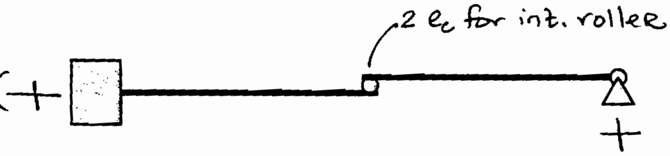
$6=6$ ∴ det. iff stable

UNSTABLE!!!



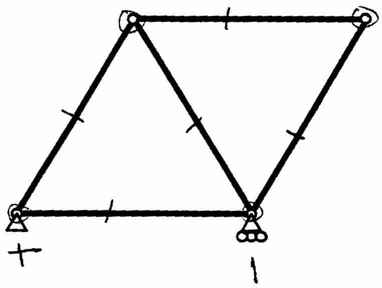
(f) UNK EQNS
8 vs 3 + 3
8 > 6 ∴ IND 2° IFF STABLE

UNSTABLE !!!



(g) UNK EQNS
5 vs 5 ∴ det iff stable.

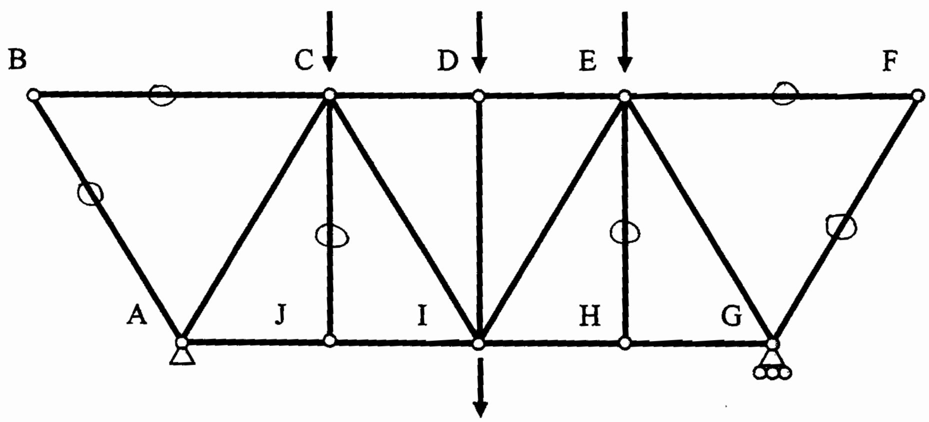
Stable ∴ Determinate



(h) b = 5 b + r $\overset{?}{=}$ 2j
r = 3 5 + 3 = 2(4)
j = 4 8 = 8
Det. iff stable

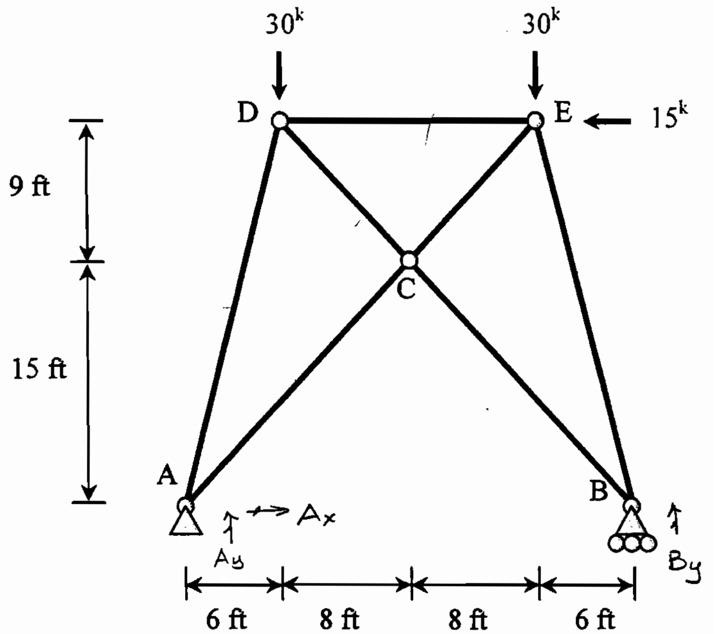
Stable ∴ Determinate

2. [9 pts.] Identify all zero force members in the following loaded truss:



Six zero-force members, as labeled.

3. [25 pts] Solve for the forces in all members of the following truss. Be sure to indicate whether each force is tensile or compressive.



Rxns

$+\uparrow \sum F_y = 0 = A_y - 30^k - 30^k + B_y$

$\overset{+}{\rightarrow} \sum F_x = 0 = A_x - 15^k$ ∴ $A_x = 15^k \rightarrow$

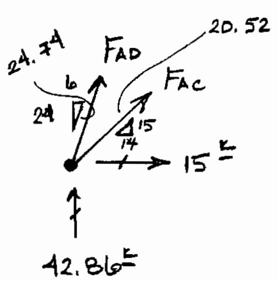
$\circlearrowleft^{+} \sum M_A = 0 = -30^k(6') - 30^k(22') + 15^k(24') + B_y(28')$

$B_y = 17.14^k \uparrow$

$A_y = 42.86^k \uparrow$

Method of Joints:

Jt A



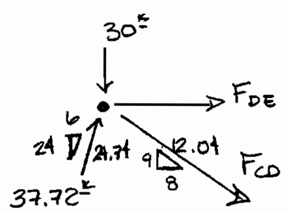
$+\uparrow \sum F_y = 0 = 42.86 + \frac{24}{24.74} F_{AD} + \frac{15}{20.52} F_{AC}$

$\overset{+}{\rightarrow} \sum F_x = 0 = 15^k + \frac{6}{24.74} F_{AD} + \frac{14}{20.52} F_{AC}$

$F_{AD} = -37.72^k$, neg ∴ C

$F_{AC} = -8.58^k$, neg ∴ C

Jt D



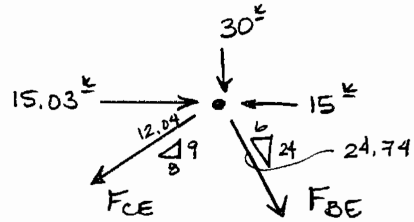
$+\uparrow \sum F_y = 0 = -30 + \frac{24}{24.74}(37.72) - \frac{9}{12.08}(F_{CD})$

$\overset{+}{\rightarrow} \sum F_x = 0 = \frac{6}{24.74}(37.72) + F_{DE} + \frac{8}{12.04} F_{CD}$

$F_{CD} = 8.85^k$, pos ∴ T

$F_{DE} = -15.03^k$, neg ∴ C

JT E



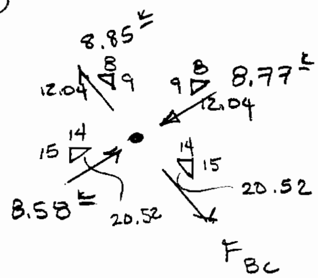
$$+\uparrow \Sigma F_y = 0 = -\frac{9}{12.04}(F_{CE}) - \frac{24}{24.74}(F_{BE}) - 30^k$$

$$\xrightarrow{+} \Sigma F_x = 0 = 15.03^k - 15^k - \frac{8}{12.04}(F_{CE}) + \frac{6}{24.74}(F_{BE})$$

$F_{CE} = -8.77^k$, neg ∴ C

$F_{BE} = -24.16^k$, neg ∴ C

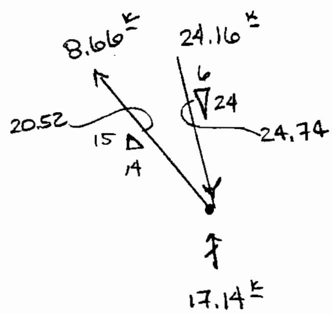
JT C



$$+\uparrow \Sigma F_y = 0 = \frac{15}{20.52}(8.58) + \frac{9}{12.04}(8.85) - \frac{9}{12.04}(8.77) - \frac{15}{20.52}$$

$F_{BC} = 8.66^k$, pos ∴ T

Check JT E :



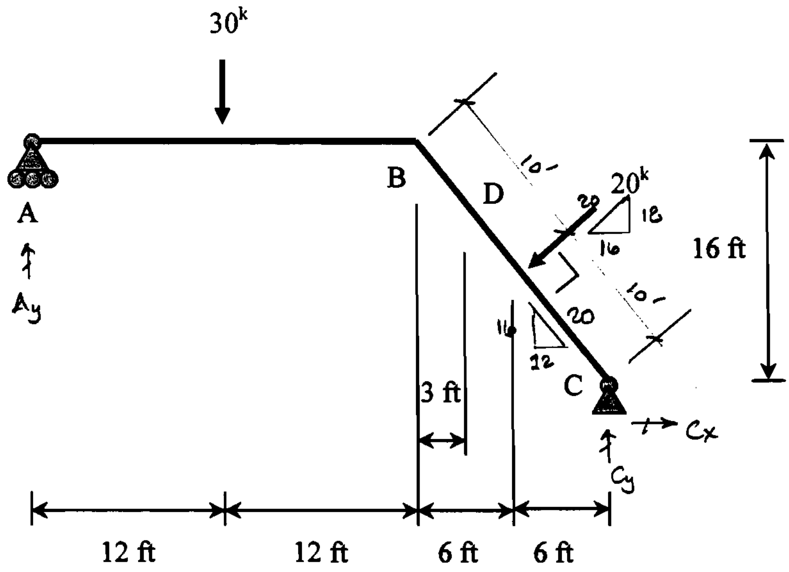
$$+\uparrow \Sigma F_y = 0 \overset{?}{=} 17.14 + \frac{15}{20.52}(8.66) - \frac{24}{24.74}(24.16)$$

$0 \overset{?}{=} 0.033^k$ OK (within Round-off error

$$\xrightarrow{+} \Sigma F_x = 0 \overset{?}{=} -\left(\frac{14}{20.52}\right)(8.66^k) + \frac{6}{24.74}(24.16)$$

$0 = -0.049^k$ OK (within R-O error

4. [25 pts] Solve for internal shear, moment, and axial thrust at point D on the frame shown below. Be sure to use positive internal force convention in your analysis.



OVERALL REACTIONS

$+\uparrow \Sigma F_y = 0 = A_y - 30^k - \frac{12}{20}(20^k) + C_y$

$\xrightarrow{+} \Sigma F_x = 0 = -\frac{16}{20}(20^k) + C_x$

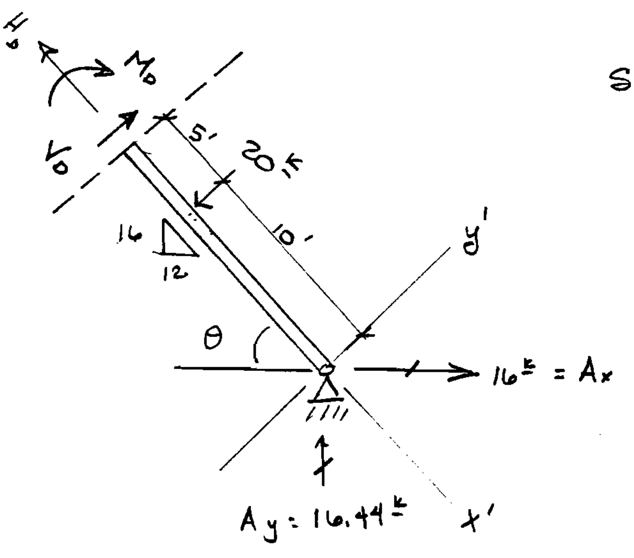
$\circlearrowleft^{+} \Sigma M_c = 0 = -A_y(36') + 30^k(24') + 20^k(10')$

$\underline{\underline{A_y = 25.56^k, \text{ pos} \therefore \uparrow}}$

$\underline{\underline{C_y = 16.44^k, \text{ pos} \therefore \uparrow}}$

$\underline{\underline{C_x = 16^k, \text{ pos} \therefore \rightarrow}}$

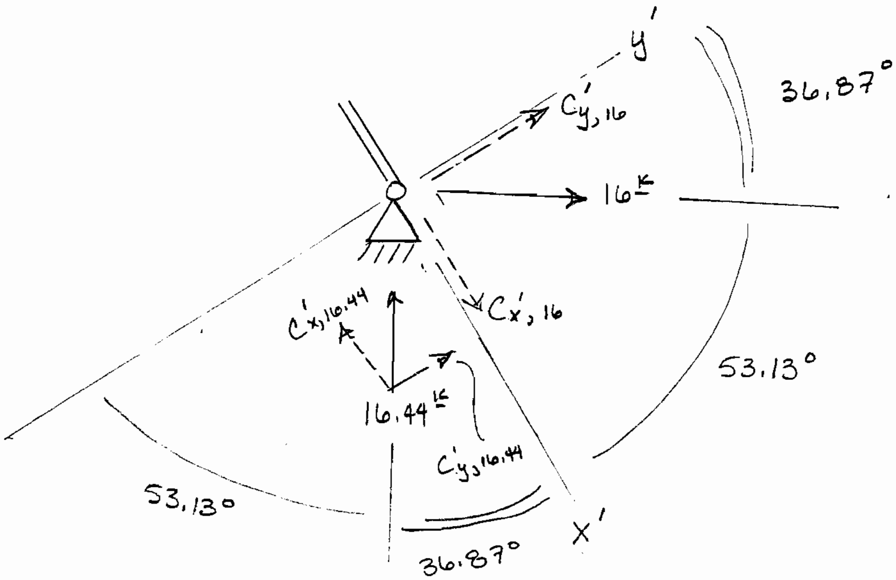
Examine FBD w/ cut at pt. D :



Solve for θ: $\sin\theta = \frac{16}{20} \therefore \theta = \sin^{-1}\left(\frac{16}{20}\right)$

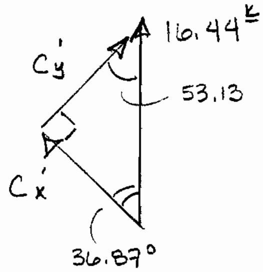
$\theta = 53.13°$

We will need to resolve A_x & A_y into A_x' and A_y' ...



Consider force triangles:

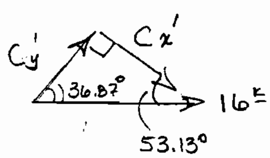
Resolve $C_y = 16.44^k$:



$\cos 36.87^\circ = \dfrac{C_x'}{16.44}$, $\underline{C'_{x,16.44} = 13.152^k}$ ↖

$\sin 36.87^\circ = \dfrac{C_y'}{16.44}$, $\underline{C'_{y,16.44} = 9.864^k}$ ↗

Resolve $A_x = 16^k$:

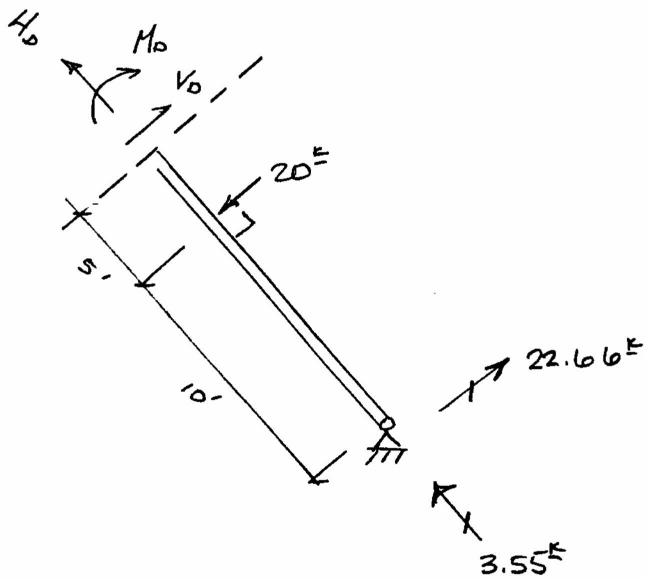


$\cos 36.87^\circ = \dfrac{C_y'}{16}$, $\underline{C_y' = 12.8^k}$ ↗

$\sin 36.87^\circ = \dfrac{C_x'}{16}$, $\underline{C_x' = 9.6^k}$ ↘

$C_y' = 9.864^k + 12.8^k = 22.66^k$ ↗

$C_x' = 13.152^k - 9.6^k = 3.55^k$ ↖

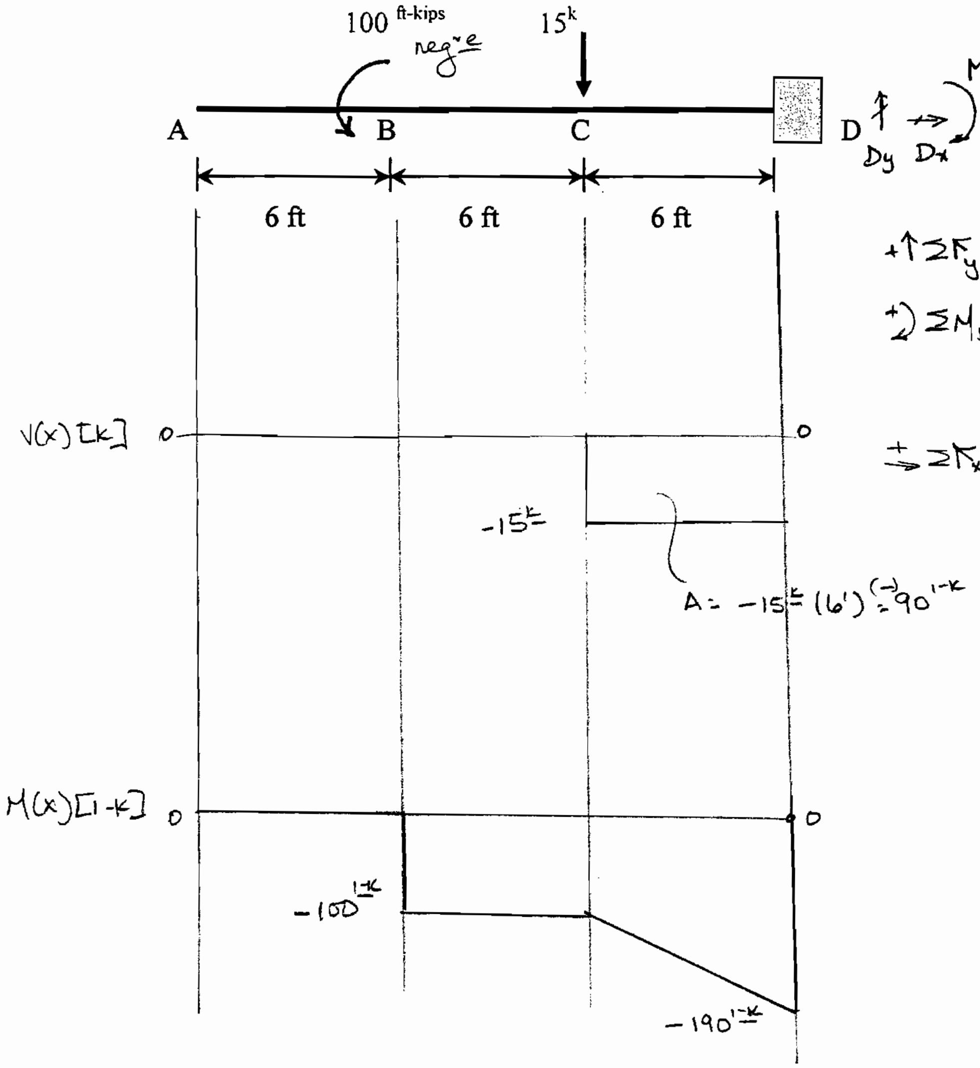


$\overset{+}{\nearrow} \Sigma F_y = 0 = V_D - 20^k + 22.67^k$ $\therefore$ $\underline{\underline{V_D = (-)\ 2.67^k,\ neg \therefore \swarrow}}$

$\overset{+}{\nwarrow} \Sigma F_x = 0 = H_D + 3.55^k$ $\therefore$ $\underline{\underline{H_D = (-)\ 3.55^k,\ neg \therefore \searrow}}$

$\curvearrowleft + \Sigma M_D = 0 = M_D + 20^k (5') - 22.66^k (15')$ $\therefore$ $\underline{\underline{M_D = 239.9^k,\ pos \therefore \curvearrowleft}}$

5. [25 pts] Draw the shear and moment diagrams for the following loaded beam. Please show your work, whether you use a graphical or equation-based approach.



Overall Rxns:

$+\uparrow \Sigma F_y = 0 = -15 + D_y \quad \therefore \quad \underline{D_y = 15^k \uparrow}$

$\overset{+}{\circlearrowleft} \Sigma M_D = 0 = M_D - 15^k(6') - 100'^{-k}$

$\underline{\underline{M_D = 190'^{-k} \circlearrowleft}}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 = \underline{\underline{D_x}} \quad 0$