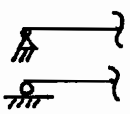
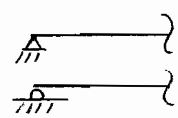
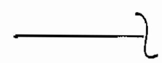
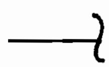
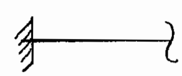
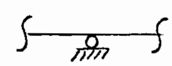
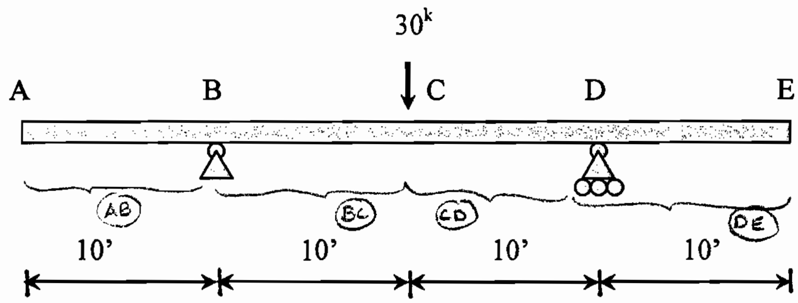
CE 461 – Structural Analysis, Fall 2006 **Name:** Caroline R. Bennett
EXAM No. 2

1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

REAL BEAM		**CONJUGATE BEAM**	
Type of Support	**Real Slope and Deflection**	**Conjugate Shear and Bending Moment**	**Type of Support**
Simple end support 	$\theta \neq 0$ $\Delta = 0$	$V \neq 0$ $M = 0$	
Fixed support (encastre) 	$\theta = 0$ $\Delta = 0$	$V = 0$ $M = 0$	
Free end 	$\theta \neq 0$ $\Delta \neq 0$	$V \neq 0$ $M \neq 0$	
Simple interior support 	$\theta \neq 0$ $\Delta = 0$	$V \neq 0$ $M = 0$	
Internal hinge 	$\theta \neq 0$ $\Delta \neq 0$	$V \neq 0$ $M \neq 0$	

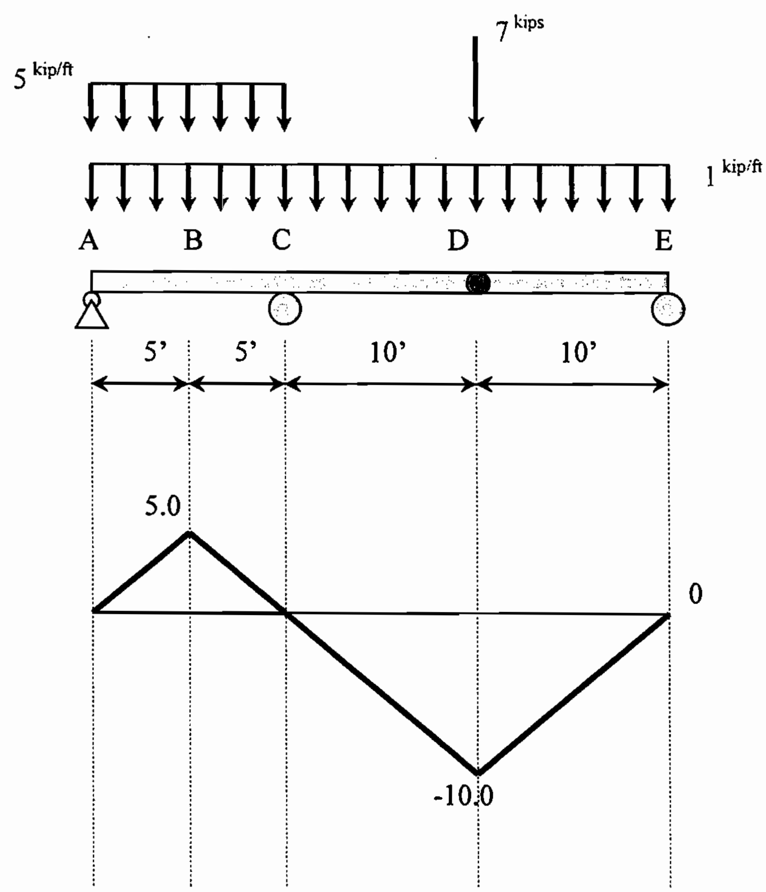
2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?



4 distinct regions –
4 constants / region

16 constants total

3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the [redacted] moment at B generated by the loads shown.



$M_B = 5^{k}/_{ft}\left[\frac{1}{2}\cdot 10\cdot 5\right]$
$\quad + 1^{k}/_{ft}\left[\left(\frac{1}{2}\cdot 10\cdot 5\right) - \left(\frac{1}{2}\cdot 20\cdot 10\right)\right]$
$\quad + 7^{k}\left[-10\right]$

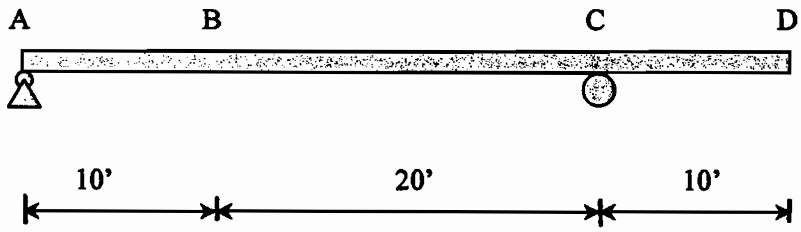
$M_B = [125^{'\text{-}k}] - [75^{'\text{-}k}] - [70^{'\text{-}k}]$

$\boxed{M_B = -20^{'\text{-}k}}$

4. [35 pts] Given the following beam, draw the influence lines for:

- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

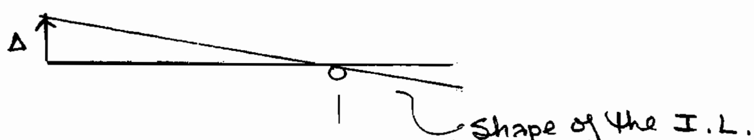
You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.



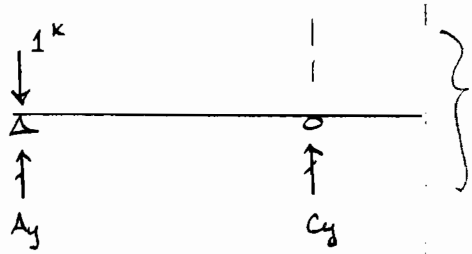
I have chosen to use Müeller-Breslau's Principle

I.L. for Vertical Reaction at A, A_y.

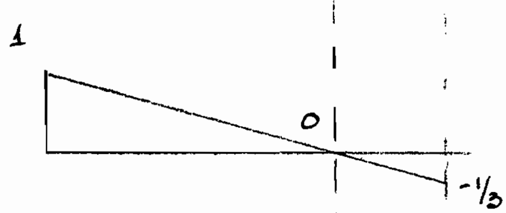
Release the vertical rxn at A & allow to deflect:



Apply a unit load at A & solve for A_y to determine the ordinate at A on the I.L.:



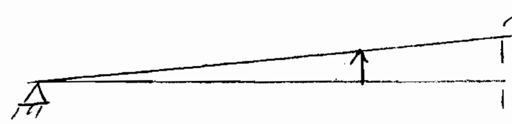
$\Sigma M_c = 0 \Rightarrow A_y = 1^k \uparrow$



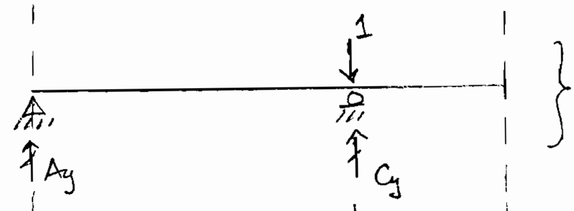
I.L. for A_y

I.L. for Vertical Rxn @ C, C_y:

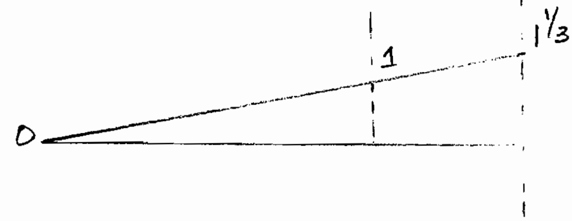
Release the vertical rxn @ C, & allow to deflect:



Apply a unit load at C & solve for C_y to determine the ordinate at C on the I.L.:



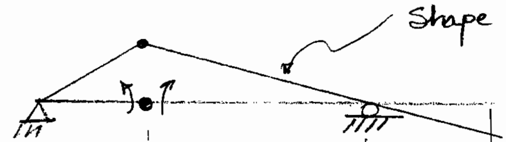
$$\Sigma M_A = 0 \Rightarrow C_y = 1^k \uparrow$$



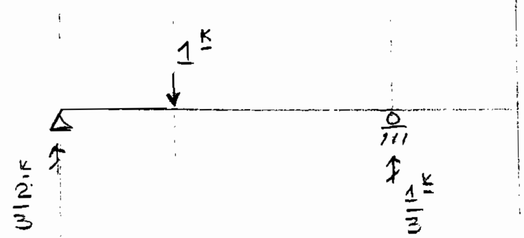
I.L. for C_y

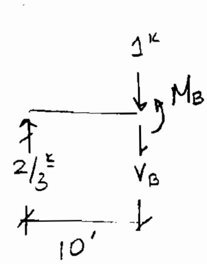
I.L. for Moment at B, M_B:

Release moment at B by inserting a hinge at B, and allow to deflect:



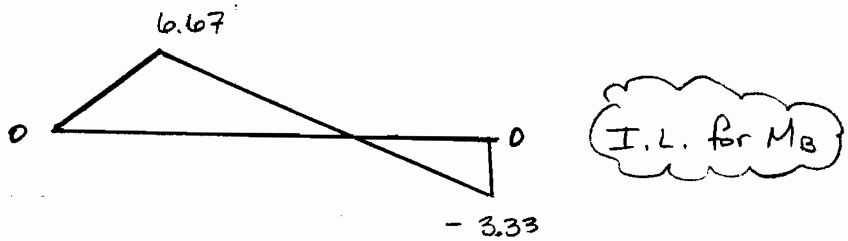
Apply a unit load at B & solve for M_B to determine the ordinate at B on the I.L.



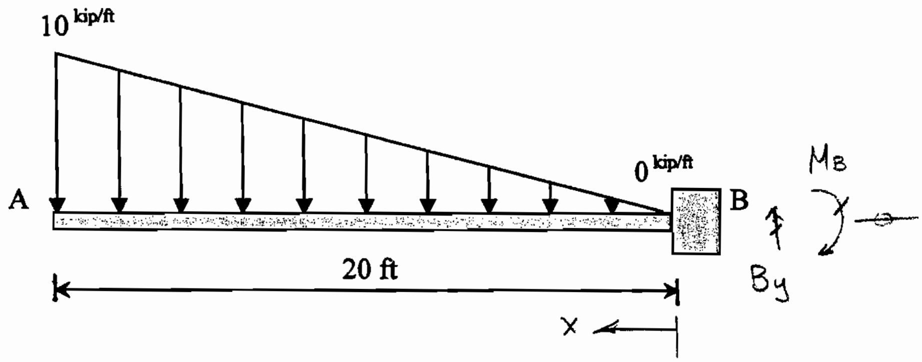


$$+\circlearrowright \Sigma M_B = 0 = M_B - \tfrac{2}{3}(10')$$

$$M_B = 6.67^k$$


6.67
0
0
-3.33
I.L. for M_B

5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let E=29,000 ksi and I=4200 in^4. Your answer should be given in inches.



Reactions:

$$+\uparrow \Sigma F_y = 0 = -10^{k/ft}(20')\left(\tfrac{1}{2}\right) + B_y \Rightarrow \underline{B_y = 100^{k} \uparrow}$$

$$+\circlearrowright \Sigma M_B = 0 = M_B - 10^{k/ft}(20')\left(\tfrac{1}{2}\right)\left(\tfrac{2}{3}\cdot 20'\right) \Rightarrow \underline{M_B = 1333.33^{1\text{-}k}}$$

Taking coordinates from the fixity to the left, $\xleftarrow[x]{} 0$

(1) $$\boxed{w(x) = \tfrac{1}{2}x}$$

(2) $$V(x) = \int -w(x)\,dx = \int -\tfrac{1}{2}x\,dx = -\tfrac{1}{4}x^2 + C_1$$

$$V(0) = 100^{k} \therefore 100^{k} = -\tfrac{1}{4}(0)^2 + C_1 \quad \therefore C_1 = 100^{k}$$

$$\boxed{V(x) = -\tfrac{1}{4}x^2 + 100}$$

(3) $$M(x) = \int V(x)\,dx = \int -\tfrac{1}{4}x^2 + 100\,dx = -\tfrac{1}{12}x^3 + 100x + C_2$$

$$M(0) = -1333.33^{1\text{-}k} \therefore -1333.33 = -\tfrac{1}{12}(0)^3 + 100(0) + C_2$$

$$\therefore C_2 = -1333.33$$

$$\boxed{M(x) = -\tfrac{1}{12}x^3 + 100x - 1333.33}$$

(4) $$EI\,\theta(x) = \int M(x)\,dx = \int -\tfrac{1}{12}x^3 + 100x - 1333.33\,dx$$

$$EI\,\theta(x) = -\tfrac{1}{48}x^4 + 50x^2 - 1333.33x + C_3$$

$$EI\,\theta(0) = 0 \therefore 0 = C_3$$

$$\boxed{EI\,\theta(x) = -\tfrac{1}{48}x^4 + 50x^2 - 1333.33x}$$

(5) $EI\,y(x) = \int \theta(x)\,dx = \int -\frac{1}{48}x^4 + 50x^2 - 1333.33\,x\;dx$

$$EI\,y(x) = -\frac{1}{240}x^5 + \frac{50}{3}x^3 - 666.67\,x^2 + C_4$$

$$EI\,y(0) = 0 \quad \therefore \quad 0 = C_4$$

$$\boxed{EI\,y(x) = -\frac{1}{240}(x)^5 + \frac{50}{3}(x)^3 - 666.67(x)^2}$$

Max deflection will occur @ the free end, where $x = 20'$.

$$EI\,y(20') = -\frac{1}{240}(20)^5 + \frac{50}{3}(20)^3 - 666.67(20)^2$$

$$EI\,y_{max} = -146,667\ \text{k}\cdot\text{ft}^3$$

$$y_{max} = \frac{-146,667\ \text{k}\cdot\text{ft}^3\ (12''/')^3}{29,000\ \text{ksi}\ (4200\ \text{in}^4)} = \boxed{-2.08'',\ \text{neg} \therefore \downarrow}$$

(This is a big deflection, but the applied load was also very large for a cantilevered beam.)