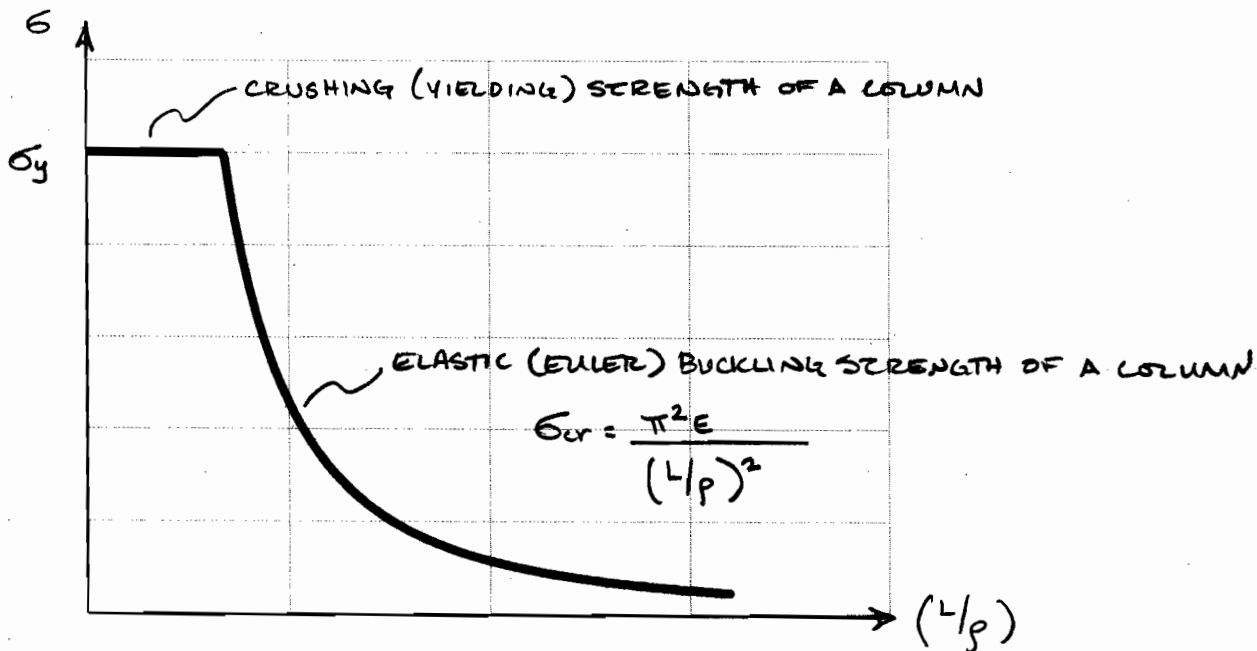


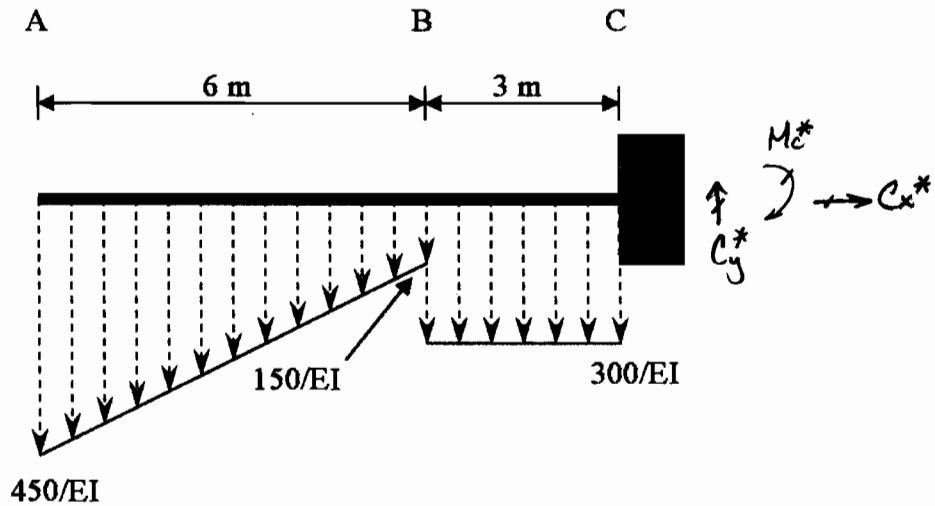
Thanks for all your hard work this semester, and best of luck in your future studies and professional endeavors. It has been a pleasure having each of you in this course. If you would like your final exam grade and your final course grade emailed to you, please indicate so here:
Y / N

1. (5 pts.) Circle the correct bolded term: A column will always buckle about the axis with the **lesser / greater** slenderness ratio.

2. (5 pts.) Describe the meaning of the following plot. Specifically, (a) label the axes, (b) identify the significance of the point at which the horizontal line intersects the ordinate (y-axis), (c) identify the significance of the horizontal portion of the curve, and (d) identify the significance of the curved function.



3. (10 pts.) Given the following conjugate beam under a conjugate load, determine the slope at point C on the corresponding real beam. Be sure to provide correct units and directional sense as part of your answer. Show all of your work to receive credit.



CONJUGATE RXNS: $+\uparrow \Sigma F_y = 0 = -\left(\frac{450+150}{2EI}\right)(6m) - \frac{300}{EI}(3m) + C_y^*$

$$C_y^* = \frac{2700}{EI}$$

SOLVE FOR SLOPE @ PT. C conj. shear @ C = actual slope at c.

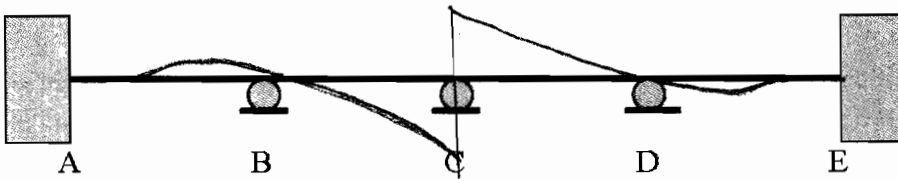
$$V_c^* = \frac{2700}{EI} \quad \int \uparrow \quad C_y^* = \frac{2700}{EI}$$

neg. shear sign convention, $\therefore V_c^* = -\frac{2700}{EI}$

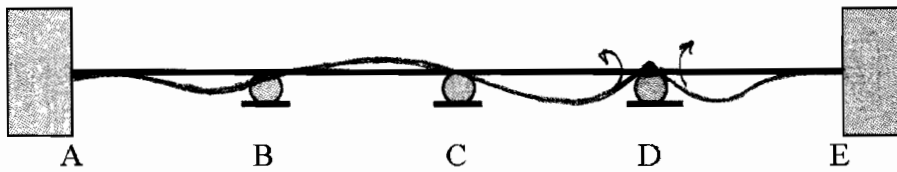
$$\theta_c = V_c^* = -\frac{2700 \text{ kN}\cdot\text{m}^2}{EI}, \text{ neg.} \therefore \leftarrow$$

4. (10 pts.) Using Mueller Breslau's principle, draw the shape of the influence lines for the following responses in the beam shown. You do not need to provide values for the ordinates, but the shapes do need to be correct.

(a) (4/10 pts. available) Shear at point C



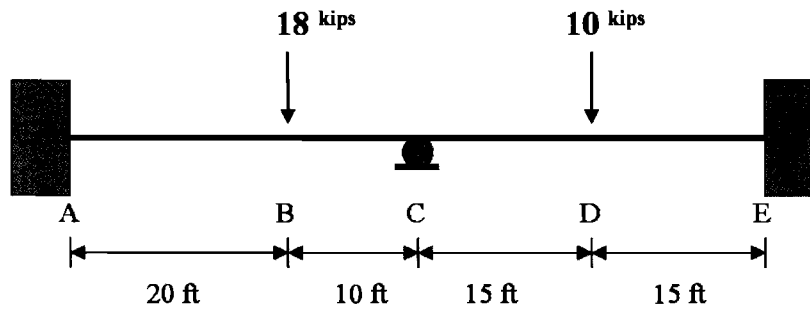
(b) (4/10 pts. available) Moment at point D



(c) (2/10 pts. available) Between what points would you place a uniform live load to cause the maximum positive moment at point D?

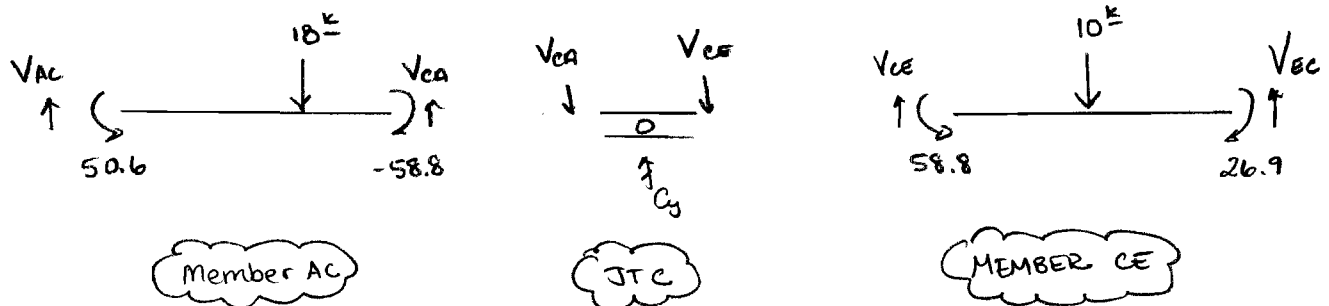
B to C

5. (10 pts.) I have analyzed the following beam for you using a stiffness method, and obtained the member end moments shown below. The member end moment signs should be interpreted using the stiffness method sign conventions. Using this information, solve for the support reaction at point C. Show all of your work to receive credit, including any complete free body diagrams that need to be considered as part of your solution.



$$M_{AC} = 50.6 \text{ ft-kips} \quad M_{CA} = -58.8 \text{ ft-kips}$$

$$M_{CE} = 58.8 \text{ ft-kips} \quad M_{EC} = -26.9 \text{ ft-kips}$$



EQUIL. OF AC : $(+\circlearrowleft) \sum M_A = 0 = 50.6 - 58.8 - 18^k(20') + V_{CA}(30')$ $V_{CA} = 12.27^k \uparrow$

EQUIL. OF CE : $(+\circlearrowleft) \sum M_E = 0 = 26.9 - 58.8 - 10^k(15) + V_{CE}(30')$ $V_{CE} = 6.06^k \uparrow$

EQUIL. OF J.T.C : $+\uparrow \sum F_y = 0 = 12.27 - 6.06^k + C_y$ $\therefore C_y = 18.33^k \uparrow$

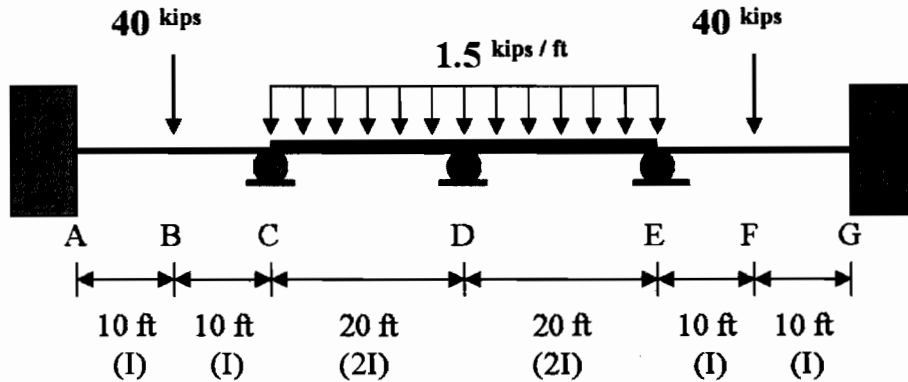
$$V_{AC} = 18 - 12.27 = 5.73^k \uparrow \quad \therefore A_y = 5.73^k \uparrow$$

$$V_{EC} = 10 - 6.06 = 3.94^k \uparrow \quad \therefore E_y = 3.94^k \uparrow$$

$$M_A = 50.6 \text{ ft-kips } \curvearrowleft$$

$$M_E = 26.9 \text{ ft-kips } \curvearrowright$$

6. (20 pts.) Determine the member end moments for members AC, CD, DE, and EG shown in the following beam. Use moment distribution. E is constant, and I varies as shown. Show all of your work to receive credit.



FIXED-END MOMENTS:

$$\begin{aligned}
 FEM_{AC} &= FEM_{EG} = +100 \text{ k}\cdot\text{ft} \\
 FEM_{CA} &= FEM_{GE} = -100 \text{ k}\cdot\text{ft} \\
 FEM_{CD} &= FEM_{DE} = +50 \text{ k}\cdot\text{ft} \\
 FEM_{DC} &= FEM_{ED} = -50 \text{ k}\cdot\text{ft}
 \end{aligned}$$

DISTRIBUTION FACTORS:

$$\begin{aligned}
 DF_{CA} &= DF_{EG} = \frac{(I/20)}{(I/20) + (2I/20)} = 0.333 \\
 DF_{CD} &= DF_{ED} = \frac{(2I/20)}{(I/20) + (2I/20)} = 0.667 \\
 DF_{DC} &= DF_{DE} = 0.50 \text{ (by inspection)}
 \end{aligned}$$

MOMENT DISTRIBUTION:

MEMBER END	AC	CA	CD	DC	DE	ED	EF	GE
DFs	—	0.333	0.667	0.50	0.50	0.667	0.333	—
FEMs	+100	-100	+50	-50	+50	-50	+100	-100
BAL		16.67	33.33	—	—	-33.33	16.67	
C.O.	8.33			16.67	-16.67			-8.33
FINAL MEM'S	108.33	-83.33	+83.33	-33.33	+33.33	-83.33	+83.33	-108.33

TABULATE VALUES :

<u>MEMBER</u>	<u>L (in)</u>	<u>A (in²)</u>	<u>N (kips)</u>	<u>n_v (kips)</u>	<u>n_v (NL/A)</u>
AC	48	2	+ 83.3 ^k	+1.33 ^k	2665.9
BC	60	2	-167 ^k	-1.667 ^k	8351.7
CD	36	2	0	+1 ^k	0
BD	48	2	-50 ^k	0	0
					<hr/>
					$\Sigma = 11,017.6$

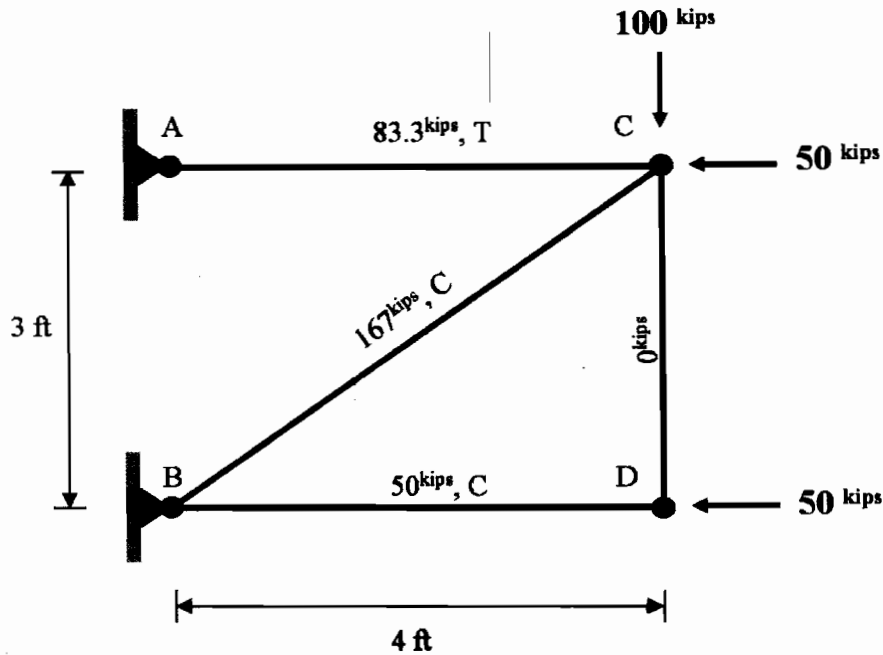
SOLVE FOR Δ_D :

$$1(\Delta_D) = \Sigma n_v \left(\frac{NL}{AE} \right)$$

$$1(\Delta_D) = \frac{11,017.6 \text{ k}^2/\text{in}}{29,000 \text{ ksi}} = 0.38'' , \text{ pos } \therefore \downarrow$$

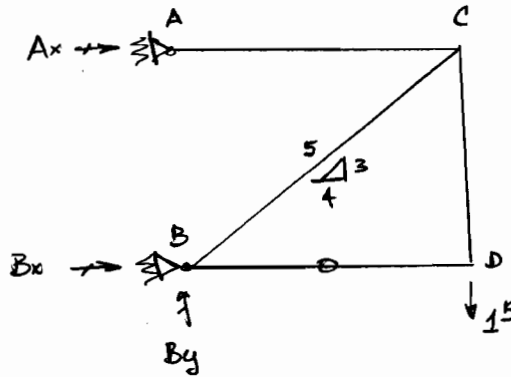
$$\boxed{\Delta_D = 0.38'' \downarrow}$$

7. (20 pts.) Determine the vertical deflection at point D using a method of your choosing. EA is constant, where $E = 29,000$ ksi and $A = 2$ in². Show all of your work to receive credit. Please note that I have provided member forces due to the loading shown in the figure.



REAL FORCE SYSTEM - FORCES PROVIDED ABOVE.

VIRTUAL FORCE SYSTEM -



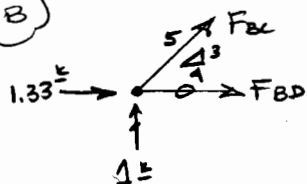
$$\sum F_y = 0 = -1 + B_y \quad B_y = 1 \text{ k} \uparrow$$

$$\sum M_B = 0 = A_x(3') - 1(4') \quad A_x = -1.33 \text{ k} \leftarrow$$

$$\sum F_x = 0 = -1.33 + B_x \quad B_x = 1.33 \text{ k} \rightarrow$$

By inspection, $F_{AC} = 1.33 \text{ k} (T)$
 $F_{CD} = 1 \text{ k} (C)$

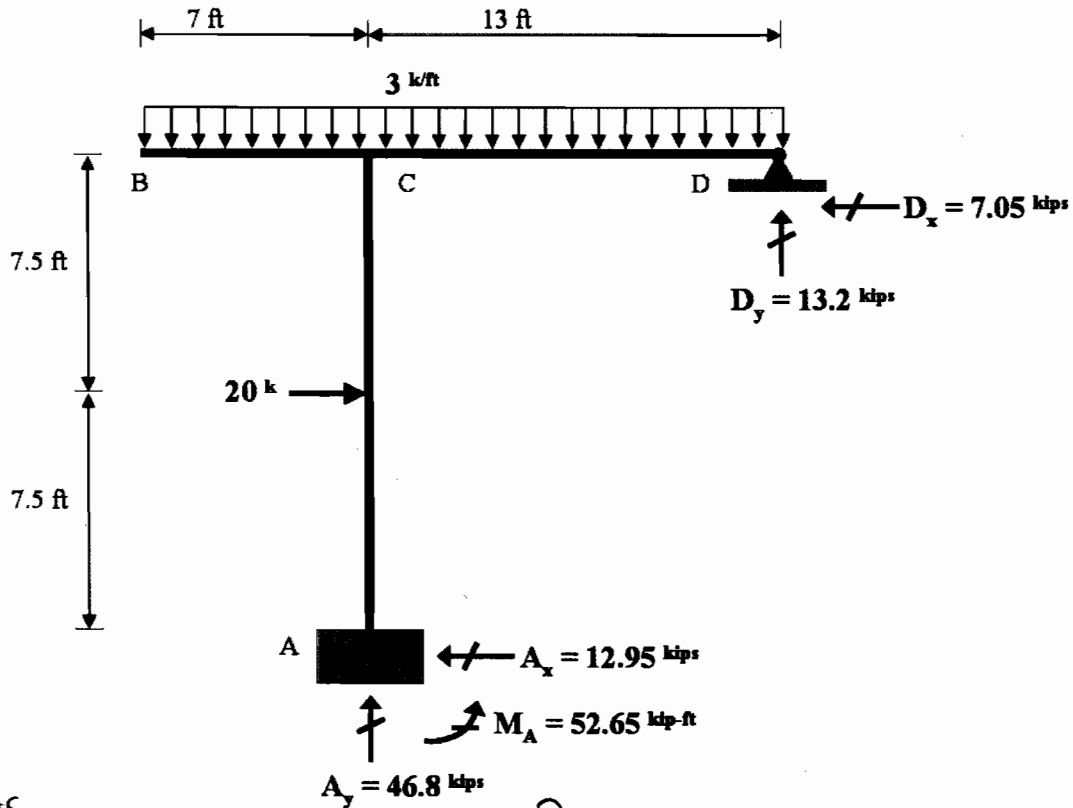
JTB



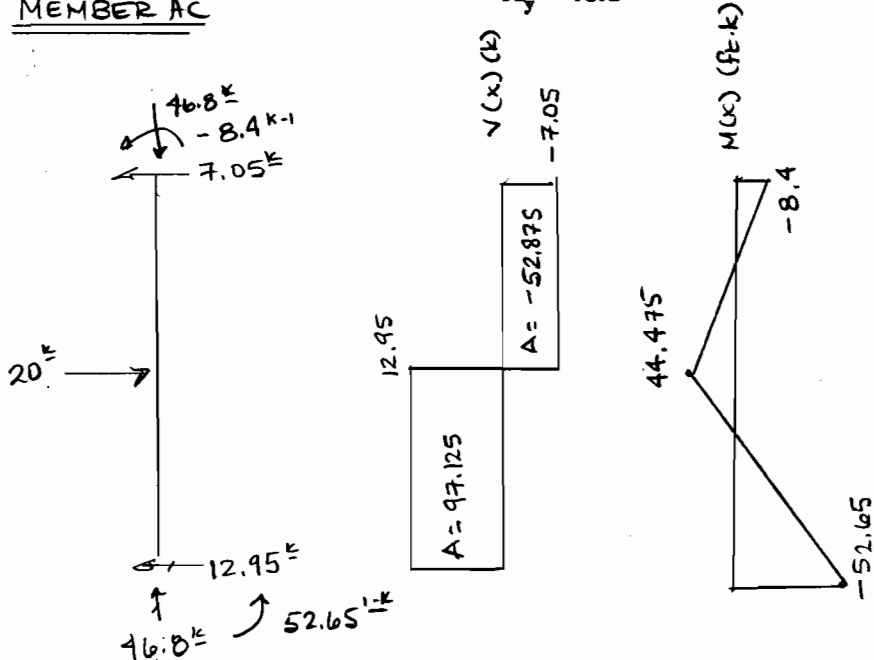
$$\sum F_y = 0 = 1 \text{ k} + \frac{3}{5} F_{BC} \quad \therefore F_{BC} = 1.667 \text{ k} (C)$$

$$\sum F_x = 0 = 1.33 \text{ k} + F_{BD} + \frac{4}{5} F_{BC} \quad F_{BD} = 0$$

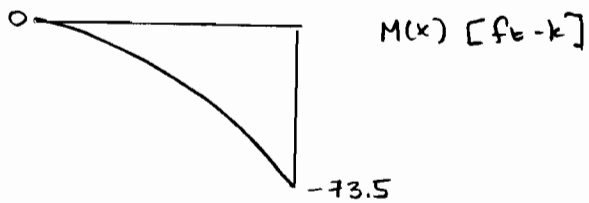
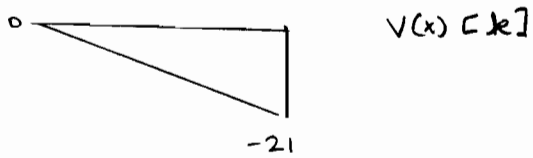
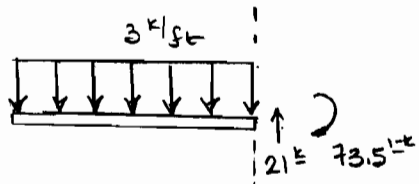
8. (20 pts.) Draw the shear and moment diagrams for the following frame. Moment should be drawn the compression side of the members. Label the maximum and minimum values on the curves. Note that I have provided all of the reactions to you. EI is constant. Show all of your work to receive credit (this includes complete free body diagrams of all the members and joint C.)



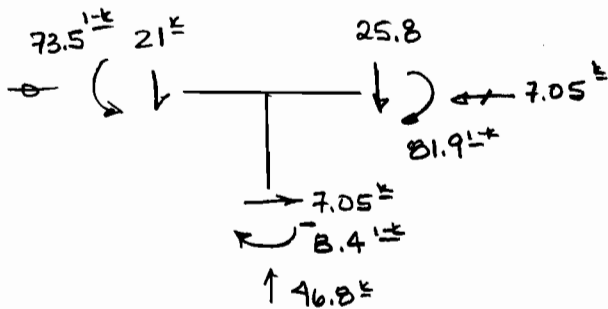
MEMBER AC



MEMBER BC



JT C EQUILIBRIUM



MEMBER CD

