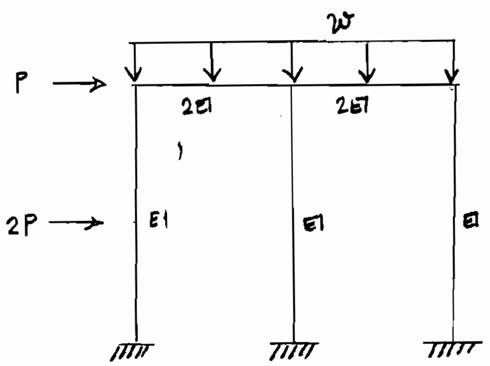
CE 461 – Structural Analysis Name: Caroline R. Bennett
Final Exam

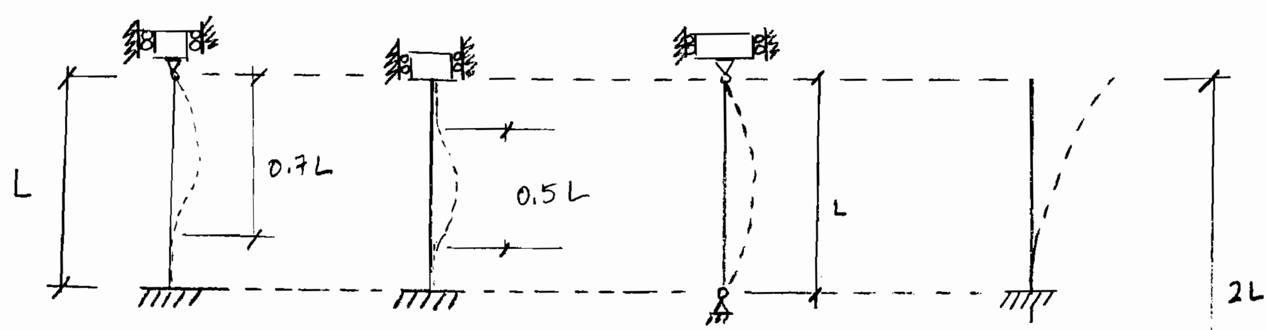
Thanks for all your hard work this semester, and best of luck in your future studies and professional endeavors. It has been a pleasure having each of you in this course. If you would like your final exam grade and your final course grade emailed to you, please indicate so here: Y / N

1. (3 pts.) Which method of indeterminate analysis that we covered in class would probably be the most *efficient* choice (least computationally intensive) to determine support reactions for the following frame? You only have a pencil, calculator, and paper as tools with which to solve the problem. Your choices are: **(1) Force Method, (2) Stiffness Method –Slope Deflection,** and **(3) Stiffness Method – Moment Distribution**. Support your answer. (You will not receive any credit without sufficient reasoning.)



I would choose slope-deflection. The force method would require calculation of a large number of deflections in order to find redundants. Since moment distribution would require two parts (w/ sidesway & w/o), it, too, would be quite lengthy. The slope-deflection method would require simultaneous sol'n of 6 eqns, which my calculator will do. If yours does not, perhaps moment distribution would be more pertinent for you...

2. (4 pts.) Please supply the effective lengths for each of the following columns.



Le = 0.70	Le = 0.50	Le = 1.0	Le = 2.0

3. (3 pts.) You have been asked by your boss to design a column of prescribed length L. The material and cross-section dimensions have been chosen for you by the architect. Your preliminary design shows that a pin-pin column of that length, material, and cross-section will buckle under the given loads. What can you do as the engineer-of-record to increase the capacity of the column without changing the overall length, material, or cross-section dimensions? Please be specific.

Change the end conditions to a more restrained case.
For example, if both ends were fixed instead of pinned, the critical buckling load would be much higher.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (\text{pin-pin})$$

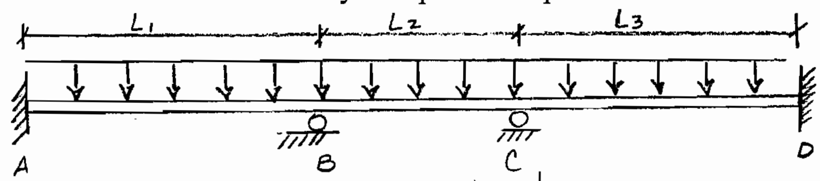
$$P_{cr} = \frac{\pi^2 EI}{(0.5L)^2} = \frac{\pi^2 EI}{0.25\,L^2} \Rightarrow$$ 4 TIMES BUCKLING CAPACITY OF A PIN-PIN COLUMN OF THE SAME LENGTH.

(Problem 4 got deleted...)

5. (3 pts.) What does a distribution factor in the moment distribution procedure physically describe?

The amount of moment each member framing into a joint will receive, depending on the stiffness of each member relative to the others.

6. (8 pts.) Fill in the following moment distribution table given the following beam, distribution factors, and fixed end moments. Carry out your precision to 0.10 ft-kip. You may not need to use all the rows in the table to successfully complete this problem.



D.F.s.		0.444	0.556	0.556	0.444	
FEMs	+78.1	-78.1	50	-50	78.1	-78.1
Bal		12.5	15.6	-15.6	-12.5	
CO	6.2		-7.8	7.8		-6.2
Bal		3.5	4.3	-4.3	-3.5	
CO	1.7		-2.2	2.2		-1.7
Bal		1	1.2	-1.2	-1	
CO	0.5		-0.6	0.6		-0.5
Bal		0.3	0.3	-0.3	-0.3	
CO	0.1		-0.2	0.2		-0.1
Bal		0.1	0.1	-0.1	-0.1	
FINAL MEMBER END MOMENTS:	86.6	-60.7	60.7	-60.7	60.7	-86.6

7. (4 pts.) For the beam in Problem 6, write the complete, numerical slope-deflection equation for M_{AB} if the support at B is subjected to a 4" upheaval due to soil swell. EI = CONSTANT
Assume L_1 is in inches, & write the equation in terms of L_1.

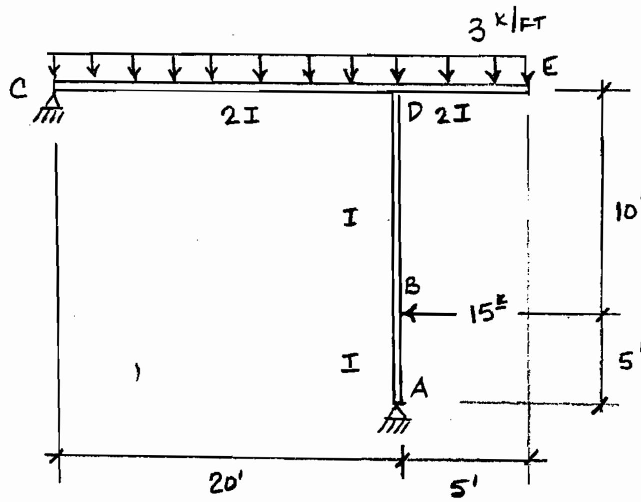
$$M_{nf} = \frac{2EI}{L}(2\theta_n + \theta_f - 3\psi) + FEM_{nf}$$

$$\boxed{M_{ab} = \frac{2EI}{L_1}\left(\emptyset + \theta_B - 3\left(\frac{+4}{L_1}\right)\right) + 78.1}$$

8. Given the following indeterminate frame,

(a) (22 pts.) Solve for the member end moments using either the **slope-deflection method** or **moment distribution method**. Please begin the problem by stating which approach you have chosen.

(b) (8 pts.)Draw the shear and moment diagrams for member AD Be sure to draw moment positive on the compression-side of the member.



E = CONSTANT

MOMENT-DISTRIBUTION

D.F.s :

$DF_{DE} = 0$ (no moment will distribute to free - fixed member).

$$DF_{DC} = \frac{\left(\frac{2I}{20}\right)}{\left(\frac{2I}{20}\right) + \left(\frac{I}{15}\right)} = 0.60$$

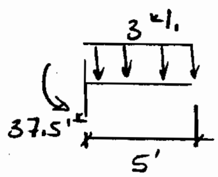
$DF_{DA} = 1 - DF_{DC} = 1 - 0.60 = 0.40$

FEMs :

$$FEM_{CD} = \frac{wL^2}{12} = \frac{3(20)^2}{12} = 100^{1\text{-}k}$$

$FEM_{DC} = (-)100^{1\text{-}k}$

$FEM_{DE} = 3^{k/l}(5')(2.5') = 37.5^{1\text{-}k}$



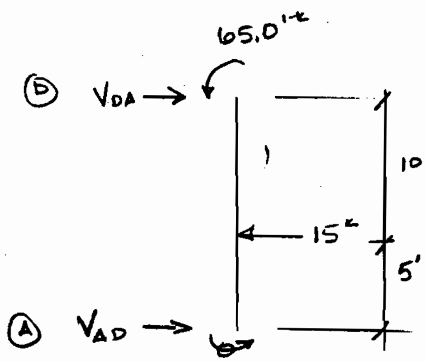
$FEM_{ED} = 0$

$$FEM_{AD} = (-)\frac{Pab^2}{L^2} = -\frac{15(5)(10)^2}{15^2} = (-)33.33^{1\text{-}k}$$

$$FEM_{DA} = (+)\frac{Pa^2b}{L^2} = \frac{15(5)^2(10)}{15^2} = (+)16.67^{1\text{-}k}$$

	MEMBER	CD	DC	DE	DA	AD
	DF	1	0.60	0	0.40	1
	FEM	+100	-100	37.5	16.7	-33.3
1)	BAL	-100	27.5		18.3	+33.3
2)	CO		-50		16.67	
3)	BAL		20		13.3	
4)	MEMBER END MOMENTS	$\boxed{0}$	$\boxed{-102.5}$	$\boxed{+37.5}$	$\boxed{+65.0}$	$\boxed{0}$

(b) V & M diagrams for member AD:

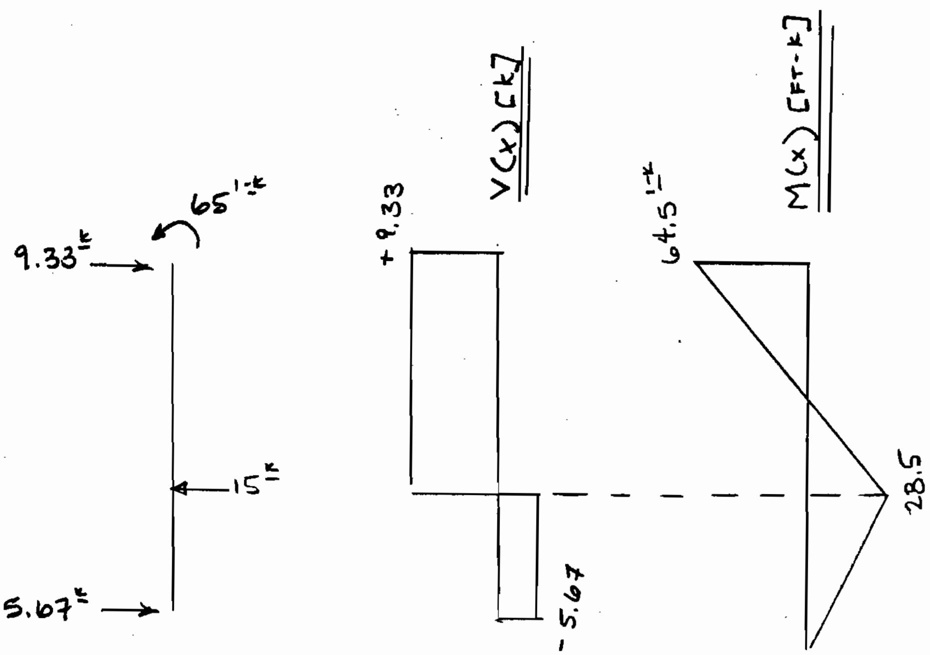


$$\circlearrowleft + \sum M_D = 0 = 65 - 15^k(10') + V_{AD}(15')$$

$$\underline{V_{AD} = 5.67^k} \rightarrow$$

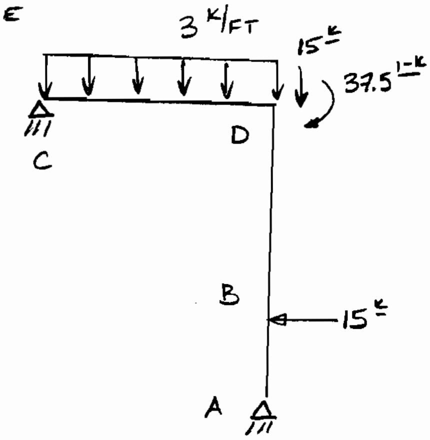
$$\xrightarrow{+} \sum F_x = 0 = 5.67^k - 15^k + V_{DA}$$

$$\underline{V_{DA} = 9.33^k \rightarrow}$$



SLOPE-DEFLECTION

SIMPLIFIED STRUCTURE



$FEM_{CD} = 100^{1\text{-}k}$

$FEM_{DC} = (-)\,100^{1\text{-}k}$

$FEM_{AD} = (-)\,33.33^{1\text{-}k}$

$FEM_{DA} = (+)\,16.67^{1\text{-}k}$

SLOPE-DEFLECTION EQNS

$M_{CD} = 0$

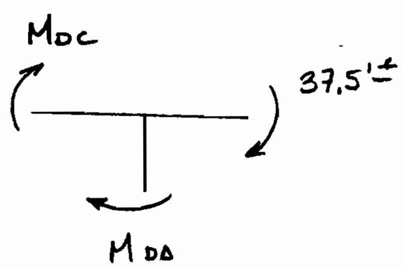
$$M_{DC} = (2)\frac{3EI}{20}\left(\theta_D - \cancel{\psi}^{0}\right) + \left(FEM_{DC} - \frac{FEM_{CD}}{2}\right) = (2)\frac{3EI}{20}\left(\theta_D\right) + \left(-100 - \frac{100}{2}\right) = (2)\frac{3EI}{20}\theta_D - 150$$

$M_{AD} = 0$

$$M_{DA} = \frac{3EI}{15}\left(\theta_D - \cancel{\psi}^{0}\right) + \left(FEM_{DA} - \frac{FEM_{AD}}{2}\right) = \frac{3EI}{15}\left(\theta_D\right) + \left(16.67 + \frac{33.33}{2}\right) = \frac{3EI}{15}\theta_D + 33.3$$

EQUILIBRIUM

JT D



$$\circlearrowleft + \Sigma M_D = 0 = M_{DC} + M_{DA} + 37.5^{1\text{-}k}$$

$$0 = \left((2)\frac{3EI}{20}\theta_D - 150\right) + \left(\frac{3EI}{15}\theta_D + 33.33\right) + 37.5$$

$$EI\theta_D = 158.34$$

MEMBER END MOMENTS

$M_{CD} = 0$

$M_{DC} = \frac{6}{20}(158.34) - 150 = (-)\,102.5^{1\text{-}k}$

$M_{AD} = 0$

$M_{DA} = \frac{3}{15}(158.34) + 33.33 = (+)\,65.0^{1\text{-}k}$

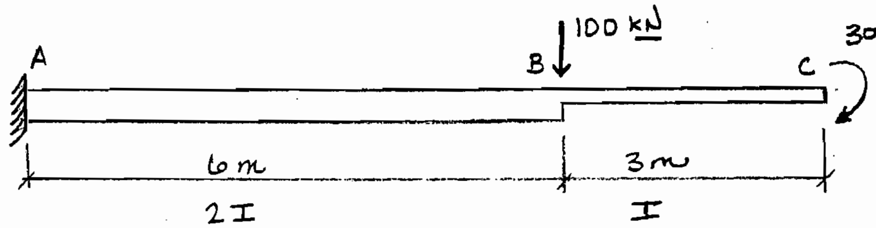
$M_{DE} = 37.5^{1\text{-}k}$

Rest of prob same as for mom-distr.

9. (25 pts.) Solve for the deflection at point C in the following beam. You may choose from one of the methods listed below.

a. **Conjugate Beam Method**

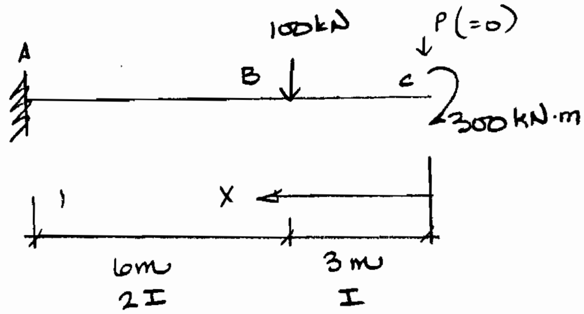
b. **Castigliano's Second Theorem (Method of Least Work)**



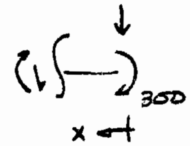
E = constant = 70 GPa

$I = 500(10^6)\ mm^4$

CASTIGLIANO'S SECOND THEOREM:



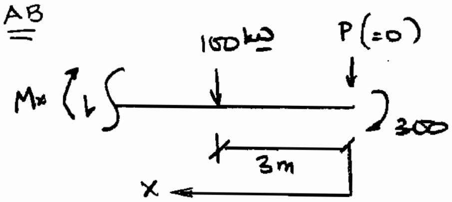
CB



$\circlearrowleft + \Sigma M_x = 0 = M_x + P(x) + 300$

$M(x)_{CB} = -300 - P(x)$

AB



$\circlearrowleft + \Sigma M_x = 0 = M_x + P(x) + 300 + 100(x-3)$

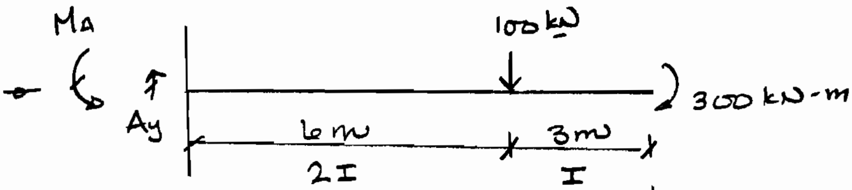
$M(x)_{BA} = -300 - P(x) - 100(x-3)$

SEGMENT	ORIGIN	LIMITS	M	$\frac{\partial M}{\partial P}$
CB	C	$0^m \to 3^m$	$-300 - Px$	$-x$
BA	C	$3^m - 9^m$	$-300 - Px - 100(x-3)$	$-x$

$$\Delta_C = \frac{1}{EI}\left[\int_0^3 (-x)(-300)\,dx + \frac{1}{2}\int_3^9 (-x)(-100x)\,dx \right] = \frac{13050\ kN\cdot m^3}{EI}$$

$$\Delta_C = \frac{13050}{70\ (500)} = 0.373\ m = \boxed{373\ mm \downarrow}$$

CONJUGATE BEAM METHOD

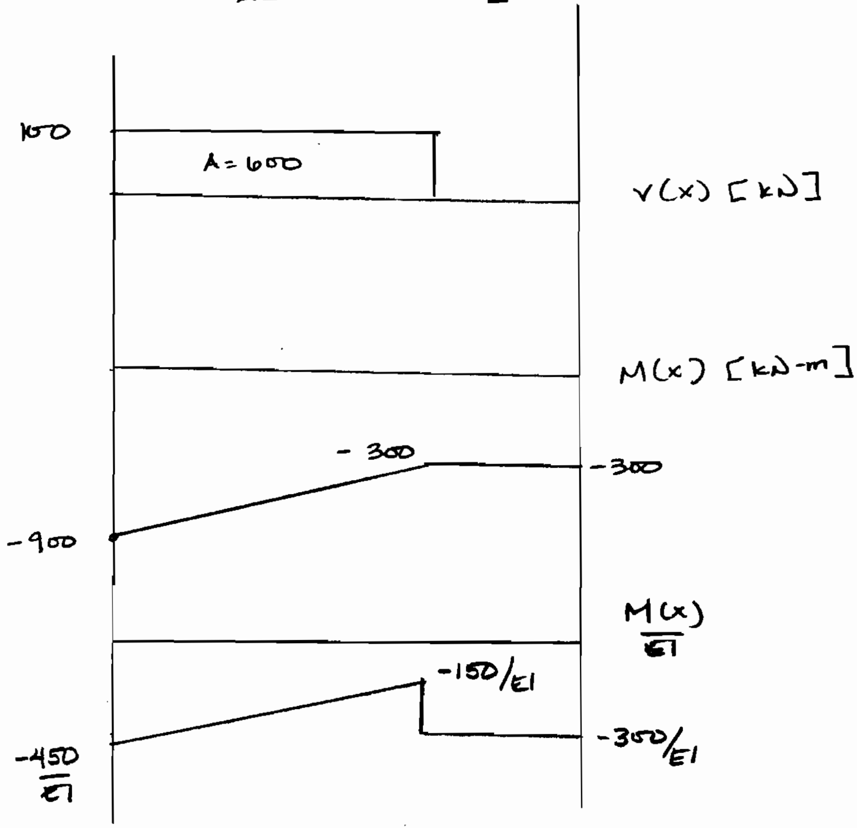


$+\uparrow \Sigma F_y = 0 \Rightarrow$

$A_y = 100\ \text{kN}$

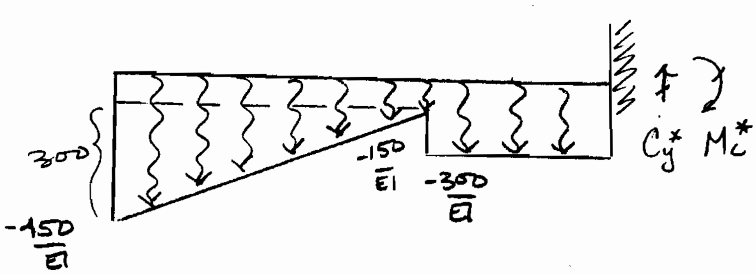
$\circlearrowleft + \Sigma M_A = 0 = M_A - 100\ \text{kN}(6\text{m}) - 300\ \text{kN-m}$

$M_A = 900\ \text{kN-m}$



Scale the M(x) diagram to a curvature diagram with constant I.

LOAD THE CONJUGATE BEAM W/ THE CONJUGATE BEAM



CONJUGATE RXNS:

$+\uparrow \Sigma F_y = 0 = \left(\frac{-450-150}{2EI}\right)(6\text{m}) - \left(\frac{300}{EI}\right)(3\text{m}) + C_y^*$

$C_y^* = \frac{2700}{EI} \uparrow$

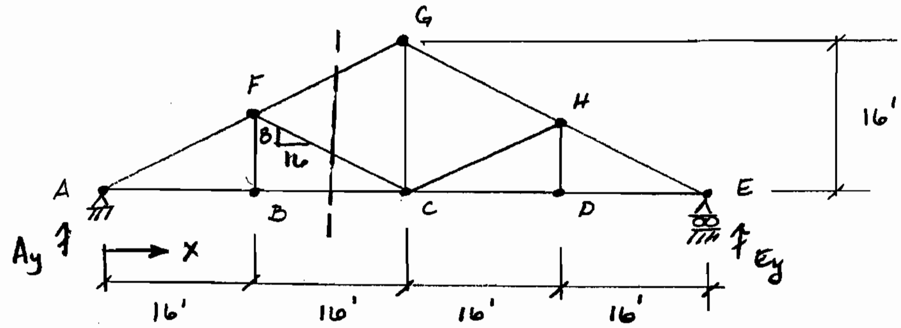
$\circlearrowright + \Sigma M_c = 0 = M_c^* - \frac{1}{2}\left(\frac{300}{EI}\right)(6\text{m})\left(\frac{2}{3}(6)+3\right) - \frac{150}{EI}(6\text{m})(6\text{m}) - \frac{300}{EI}(3\text{m})(1.5\text{m})$

$M_c^* = \frac{13050}{EI} \circlearrowright$

CONJUGATE MOMENT, M_c^*, = deflection at C.

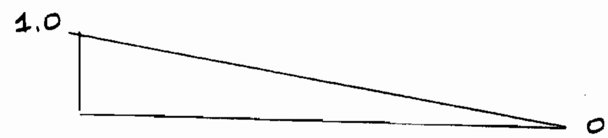
$$y_c = M_c^* = \frac{13050}{EI} = \frac{13050}{(70)(500)} = 0.373\ m \quad \therefore \quad \boxed{373\ mm \downarrow}$$

10. (20 pts.) Draw the influence line for the force in member CF for the following determinate truss.



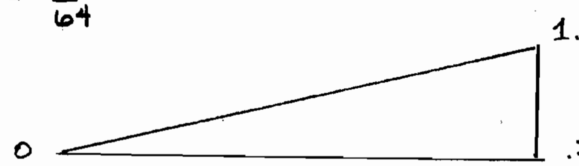
FIRST FIND I.L.s FOR RXN FORCES:

$$\circlearrowleft^{+} \Sigma M_E = 0 = -A_y(64) + 1(64-x) \Rightarrow A_y = \frac{64-x}{64} = 1 - \frac{x}{64}$$



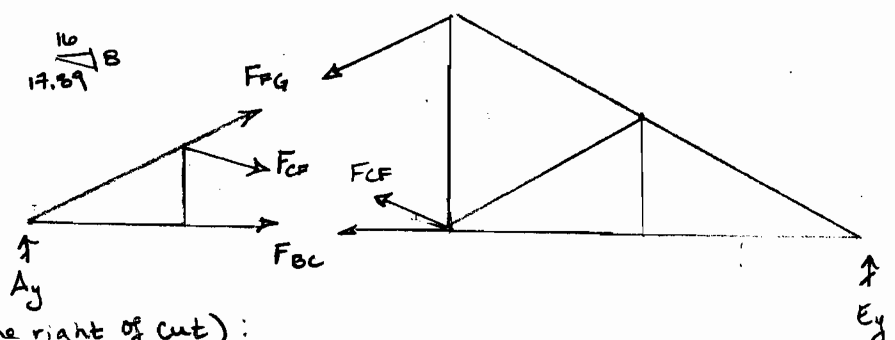
I.L. for A_y

$$\circlearrowleft^{+} \Sigma M_A = 0 = E_y(64) - 1(x) \Rightarrow E_y = \frac{x}{64}$$



I.L. for E_y

FIND I.L. for MEMBER CF



(1^k load to the right of cut):

$$\circlearrowleft^{+} \Sigma M_A = 0 = (-)\frac{16}{17.89} F_{CF}(8') (-) \frac{8}{17.89} F_{CF}(16') \Rightarrow \underline{F_{CF} = 0} \quad \text{for } 32' \le x \le 64'$$

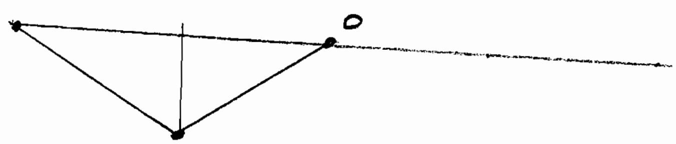
(1^k load to left of cut):

$$\circlearrowleft^{+} \Sigma M_A = 0 = \frac{8}{17.89}(F_{CF})(32') + E_y(64')$$

$$0 = 14.31 F_{CF} + E_y(64') \Rightarrow \underline{F_{CF} = -4.4725 E_y} \quad \text{for } 0' \le x \le 16'$$

$$\text{I.L. for } F_{CF} = \begin{cases} -4.4725\,E_y & 0 \le x \le 16 \\ \text{STRAIGHT line} & 16 \le x \le 32 \\ 0 & 32 \le x \le 64 \end{cases}$$

$$E_y = \frac{x}{64} \quad \therefore \quad \begin{cases} -4.4725\left(\frac{x}{64}\right) & 0 \le x \le 16 \\ \text{straight line} & 16 \le x \le 32 \\ 0 & 32 \le x \le 64 \end{cases}$$



$$-4.4725\left(\frac{16}{64}\right) = \underline{\underline{-1.12}}$$

I.L. for F_{CF}